Patent Assertion and the Rate of Innovation

Jorge Lemus[†] and Emil Temnyalov[‡]

October 31, 2013

Abstract

We analyze the effect of a particular business model of patent assertion, used by some non-practicing entities (NPEs), on the incentives for innovation. We study producing firms which engage in simultaneous patent races, in a setting with strong but probabilistic patents, where the final product uses multiple separately patentable components. We characterize the equilibrium of a model that incorporates patent trade, licensing and litigation for a given allocation of patents. We then endogenize the firms' patent portfolios as the outcome of a multi-patent race, in order to explicitly study the incentives for innovation. We show that the impact of an NPE on producing firms is two-fold. First, it increases the marginal value of patenting a discovery and thus enhances the incentives to invest in R&D, because it extracts surplus from firms with smaller portfolios. Second, in some cases it effectively acts as an entry deterrent.

WORKING DRAFT

Keywords: Innovation, R&D, Patent Policy, Non-practicing Entities.

1 Introduction

The market for intellectual property, and in particular patents, has changed significantly in recent years. First, the traditional notion of one-invention-one-patent is obsolete in many industries, particularly in the technology sector. When one product is protected by many complementary patents, the value of a

---

[†]Northwestern University. email: jlemus@u.northwestern.edu
[‡]Northwestern University. email: emil@u.northwestern.edu

patent in isolation might differ from its value inside a patent portfolio (Gans and Stern (2010)). Second, new actors have appeared in the arena and are shaping the way intellectual property markets work, by exploiting the uncertainty of litigation outcomes. Patent transactions have moved from a system where the majority of sales and licensing deals occurred between inventors and producing firms, to one where intermediaries, who do not directly innovate or participate in the downstream market, have some of the world's largest patent portfolios. These intermediaries are an entirely new kind of actor in the intellectual property sphere and have recently received a considerable amount of public attention. There is currently no consensus on the impact of such intermediaries on the economy.

In this paper we consider what are typically called pure patent assertion entities, nonpracticing entities, patent dealers or, pejoratively, "patent trolls". Specifically, we study entities which generally do not invest in R&D and do not produce products that rely on their patent portfolios. Instead, they buy existing patents from firms, inventors, and universities, generating revenue by licensing them to producing firms, under the threat of suing alleged infringers. We adopt the definition of Non-Practicing Entity (NPE) as in Hagiu and Yoffie (2013):

" ... nonpractising entities act as arbitrageurs, first acquiring patents, ... and then seeking licensing revenues from operating companies through litigation or threat of litigation. These entities do not innovate themselves, nor do they produce output."

The term non-practicing entity seems most appropriate to describe such agents—it is descriptive and neutral—although it might be interpreted to also include universities or individual inventors who do not commercialize their own inventions. Our definition explicitly excludes both, because they directly engage in innovation.

In reality there is no single entity that exemplifies all NPEs. These firms use a large array of different business models: they buy different kinds of patents and source their portfolios from different kinds of inventors, they employ different bargaining, licencing and litigation strategies. We do not attempt to study all of these different business models. Rather, we focus on a particular kind of NPE, which have been called *pure* patent assertion entities (PAE) in Scott Morton and Shapiro (2013). We isolate the impact of one specific NPE business model and study how patent trade involving such an NPE affects the rate of innovation. There is no doubt that NPEs in general play a complex role in the current system, affecting the equilibrium through a number of different channels. Incorporating all of these arguments in a single formal model is challenging, especially if we want to draw economic insights that might help the discussion of the role of NPEs in the system.[1] Instead, we study a model which includes, in our opinion, the most important features of the modern patent system, and which is rich enough to allow us to talk explicitly about the ultimate equilibrium effects on the incentives for innovation.

We propose a theoretical model to study the effects of NPEs on licencing, litigation, and, most importantly,

---

[1]Moreover, the implications of some of the arguments that have been put forth can be easily understood without a formal model, and they can easily be incorporated in one. See, for example, McDonough (2006).

on innovation, which we think is necessary in order to evaluate their role in the economy. In contrast, much of the public debate has so far been driven by appeals to what seems good or bad on the basis of vague ideas of how the patent system should operate. To us, as economists, it seems clear that the issues surrounding NPEs should be studied from the perspective of how they affect incentives, and with an eye to the optimal design of the patent and litigation systems.

A preview of our results

Our contribution to the literature is to explain the existence of NPEs in a frictionless environment and discuss their effect on the rate of innovation. We first explain the mechanism through which an NPE can make positive profit in the market, and then provide a formal exposition of its effect on the incentives for R&D. It has been argued that because NPEs extract revenue from producing firms, they must be harming innovation. We show that, even when it is true that NPEs extract surplus from producing firms, they could have a positive effect on innovation under certain conditions. This is because once the research stage is endogenized, forward looking firms will exert more effort to have a larger portfolio and capture the increased marginal value of patenting a discovery. As a consequence of the equilibrium behavior of firms, the innovation rate increases.

We show that, under certain conditions, NPEs decrease the surplus that producing firms receive as a result of their R&D, as one might have expected. But, perhaps counter-intuitively, they do so in a way that actually increases the incentives to exert research effort, and therefore NPE activity leads to a higher rate of innovation in the economy. Specifically: by buying existing patents, NPEs induce an equilibrium where the patent holders sell licences under the threat of increased litigation, which is only credible because of the presence of NPEs. This translates into lower profits for "follower" firms with smaller patent portfolios,[2] and hence the incremental value of patenting a discovery (and therefore the return to R&D effort) increases. Thus we explicitly show that the idea that NPEs are bad because they extract rents from producing firms is generally incomplete and overly simplistic. Incorporating an endogenous research stage allows us to show precisely how changes in the continuation game drive innovation incentives.

On the other hand, we also show that under some conditions NPEs may effectively act as an entry deterrent, by allowing a dominant firm to leverage its patent portfolio and block entry by competitors into the final product market. In the latter case the incentives for innovation also increase, just as in the former case, but the effect of the NPE on overall welfare is likely to be negative, due to the increased likelihood of monopolization. Overall, the main contributions of this paper are: to carefully study a particular kind of NPE business model; to show that the effect of NPEs in the patent system might be positive, even when they extract surplus from producing firms; to expose the incompleteness of some commonly heard arguments about patent acquistions and patent privateering.

---

[2]And, moreover, NPE activity only matters in cases where the producing firms have asymmetric portfolios.

Some key features of our model

The first challenge to a theoretical model of NPEs is to define an environment in which NPEs have a reason to exist, i.e. one where they can make profits in equilibrium. This can certainly be accomplished in a model where the NPE has some exogenously given (and thus somewhat artificial) advantage over the typical producing firm. One could consider a model of litigation where NPEs have lower legal costs, and therefore alter the equilibrium of the patent system; such lower costs can be taken as a maintained assumption or derived through some form of economies of scale in litigation, but in either case it is not clear why a producing firm would be unable to replicate exactly the strategy of an NPE, for instance by employing its patent management division as if it were an NPE. Alternatively, one could consider a model where firms search through a pool of patents in order to establish which ones their products might infringe upon, or what the prior art is for a new invention, and consider the case where NPEs have lower search costs (Biglaiser (1993)). Rather than endowing the NPE with some inherent advantage, we will consider a model where all entities are identical in terms of their litigation and licensing opportunities and show that NPEs can indeed make positive profits in equilibrium and affect the outcomes of producing firms. In other words, we can explain the existence of NPEs in a model without any technological frictions, and thus the insights of this paper are more transparent and intuitive.

Our model captures some of the key features of the current state of the patent system. First, our model considers trade in strong, but probabilistic patents. They are strong in the sense that litigation against a firm which infringes on a patent would still be profitable after taking into consideration legal costs. We do not model patents whose litigation value would be negative.[3] Notice that in general a patent may have negative value in a one-shot game and yet have positive value in a repeated game, because the patent holder may want to establish a reputation for enforcing such patents.[4] In our model litigation is a one-shot interaction and the threat of litigation is credible. Hence we do not attempt to study the issues of "weak patents" or what one might call "spurious invention", as other papers have explored. However, our main intuition could be embedded into a model that also incorporates the patent thicket problem.

Second, our model makes no distinction between "large" and "small" firms. We simply model a producing firm as an entity which invests in its own R&D, obtains patents on its inventions, and has the option to licence or sell its patents, and to litigate against alleged infringers. An NPE, on the other hand, is a firm which has no capacity to develop an invention or to produce a final good, but may trade in patents and engage in licencing and litigation on the basis of its portfolio. We study licencing in the shadow of litigation. That is, when a patent holder offers a producing firm a licence, all parties will rationally take into consideration what their actual payoffs would be if they do not agree on a licence price. Therefore the licence that they agree on may reflect some counter-factual litigation costs, and not just the pure value of the invention itself.

---

[3]A patent may have negative litigation value because the legal cost of litigation outweighs the expected value of the rewards that the patent holder would receive from the lawsuit.

[4]See Scott Morton and Shapiro (2013) and Hovenkamp (2013) for a discussion of such strategies employed by some NPEs.

Third, in many industries a single patent does not map into a product–this is very much the case in the technology sector, semiconductors, smartphones, etc. This leads to much higher interdependence among producing firms which make products that rely on overlapping pieces of technology, and increases the importance of licensing and litigation among them. Moreover, the complexity of a product and the fact that it relies on many patents held by different entities creates the incentive for a firm to trade in patents strategically, in order to defend itself against potential infringement suits or to initiate such against its rivals. Thus, the *market* for patents itself is more important today than ever–it matters both because of the traditional role it has served in efficiently allocating who should commercialize an invention, and because of its role as an exchange where firms trade strategically in order to exploit arbitrage opportunities in intellectual property. We therefore study a model where firms patent separate components that are necessary to make a product, rather than patenting an entire product as in Lemley and Shapiro (2007).

Fourth, the complex nature of products and innovations makes it hard to determine with certainty what should be patentable and what should not, and patents are often granted by the patent office and later invalidated in court. As Lemley and Shapiro (2005) point out, patents are now essentially "probabilistic property rights", or "lottery tickets", with many of them being invalidated in court. Moreover, determining whether a particular product infringes on a patent, even if it is a valid patent, is not straightforward. The boundaries of particular innovations are often blurred (and sometimes deliberately so by the original patent claimant, who may not want to be transparent in terms of what exactly a patent covers), which generates significant uncertainty as to whether a patent would be found relevant in a lawsuit against an alleged infringer. As in Lemley and Shapiro (2007), this paper explicitly considers the outcome of any infringement suit as a probabilistic function of the plaintiff's patent portfolio, to capture the unpredictability of litigation.

Fifth, our primary goal in this paper is to characterize the effect of a new actor in the intellectual property sphere, NPEs, on the incentives for research and development, which may accrue not just from the commercialization of a patent as part of a product, but also from licensing or litigation. We therefore embed our model of patent trade, licensing, and litigation, into an explicit larger model of the research efforts and investments of competing firms. In other words, we study the process of trading, licensing and suing as a continuation game that follows what is essentially a multi-stage patent race involving several components that are all necessary for the production of a final good. We thus have a formal model of the incentives that drive innovation, and can discuss how the benefits to invention will determine what firms are willing to invest in research, and how this will in turn determine the equilibrium rate of innovation in the economy. This allows us to characterize the effect of NPEs, by comparing the economy's rate of innovation in the benchmark case where they do not exist (or, hypothetically, are not allowed to enforce patents in the same way that a practicing entity is), to the case where they do exist.

Finally, one NPE practice that has been discussed as detrimental to innovation is *patent privateering*[5], whereby a producing firm sells part of its portfolio to an NPE and lets it enforce the patents. One reason

---

[5]See Ewing (2012) for an introduction to the topic.

that one might consider privateering detrimental is because an NPE would never cross-license with a producing firm, which could break down a cross-license equilibrium between two producing firms. A firm with a large patent portfolio might obtain a small marginal value from a small part of its portfolio. In that case, selling a small part of its porfolio to an NPE might be optimal from the firm's perspective. A second practice recognized as detrimental is the disaggregation of a patent portfolio, which can generate a *royalty stacking* problem, whereby an alleged infringer has to pay licenses multiple times because a product infringes on patents owned by different parties. Since damages awards are not easily determined, the infringer might be paying more than it should in royalties. We incorporate both of these effects in our model, and while it is true that both practices extract surplus from producing firms with smaller portfolios, we show that this does not necessarily imply a lower rate of innovation.

What comes next

In the next section we discuss why NPEs matter in the patent and litigation system and review some of the literature and the arguments that have been offered for and against NPEs. In section 3 we set up the model of patent trade, licencing and litigation, present the assumptions that we will use in the analysis, and describe the endogenous research stage. Section 4 considers an economy where NPEs do not exist, while Section 5 presents the equilibrium of the model with an NPE. Section 6 contains our main results on patent trade, the impact of an NPE on innovation, and the possibility of entry deterrence. Section 7 presents some extensions of the model. Section 8 concludes. The detailed analysis of equilibrium payoffs is found in Appendix A, while Appendix B contains the proofs of our main results. Appendix C presents a further robustness extension of our model, and Appendix D contains some technical details of the extensions in the text.

2 Institutional Details and Related Literature

NPEs are clearly very important in intellectual property markets: the number of operating companies involved in NPE lawsuits has been growing at an average annual rate of 36% since 2004, increasing from 636 in 2004, to 3859 in 2012.[6] Although such litigation often produces fairly small rewards, ranging between $50,000 and a few million dollars, some of the most well-known cases have resulted in payments in the hundreds of millions of dollars (Hagiu and Yoffie (2013); Sharma and Clark (2008)). Moreover, NPEs also generate significant revenues by selling licenses. Although license prices are often secret, Bessen and Meurer (2012) estimate $1.33 million as mean settlement costs for small/medium companies and $7.27 for large companies. The increased concern about NPEs by regulators and Congress is made clear in a series of Acts: the SHIELD Act, the Patent Quality Improvement Act, America Invents Act and the End Anonymous Patents Act.

---

[6] According to PatentFreedom, https://www.patentfreedom.com/about-npes/litigations/. See also Chien (2008).

Whether the new patent environment that has emerged is beneficial or detrimental for innovation has been the subject of much academic and public debate. The patent system is designed with the goal of encouraging innovation by guaranteeing rewards to inventors, so it is natural to evaluate its performance in terms of the incentives for innovation. It is also worth noting that the system was not designed so that only practicing entities would receive patent protection (Denicolò (2007); Mazzeo et al. (2011)).

Arguments for and against

With that in mind, we can now summarize some of the main arguments against NPEs. First, NPEs tend to engage in licensing and litigation with established companies that have been selling a product for some time, and often at a point in time when the company's product is most vulnerable, e.g. after irreversible investments in a product have been made, since that is when a patent that the product relies on is most valuable. Second, some have argued that NPEs, by litigating and licensing patents that a company is unaware of, raise the costs of operating a firm, since the latter now has to invest more in finding out what patents exist that its product may be infringing upon, or what the state of prior art is for its own patents. Third, NPEs have been accused of engaging in "nuisance litigation", whereby an entity litigates on the basis of weak patents, purely for the purpose of threatening to impose legal fees on the defendant, so that it may instead choose to settle early on (Scott Morton and Shapiro (2013) & Hovenkamp (2013)). NPEs are therefore accused of exploiting the uncertainty that is inherent in many patents, especially in industries where the boundaries of a patent may be fuzzy(Agliardi and DSE (2009); Miller (2012)), and where patents have been dubbed "lottery tickets" (Lemley and Shapiro (2005)). Fourth, to the extent that an NPE extracts some fraction of the overall industry surplus as rents, without increasing the size of the total surplus, the NPE may decrease the rewards to innovation and hence weaken the incentives of inventors (Bessen and Meurer (2012)).

On the other hand, NPEs may enhance innovation if they reward inventors who would otherwise not be able to enforce their patents due to financing or litigation frictions. If an individual inventor cannot reap the rewards of his efforts due to some imperfection of the system, such as the lengthy and costly nature of trials, then an NPE could potentially buy the patent and either enforce it in litigation, or extract a licensing fee under the threat of litigation–some of which may be passed down to the inventor, thus rewarding innovation in the way that the patent system was originally designed to. Second, NPEs may be good for innovation if they provide valuable liquidity to the market for patents, in the same way that a stock broker provides liquidity to a financial market. Patents are especially hard to value, since by definition they are fairly unique and lack close substitutes whose market valuation could be used as a proxy for the value of any particular patent. Moreover, they are often traded and licensed privately, and hence even if a patent is traded, its value may not be observable by an outside party. So to the extent that NPEs increase the number of trades in the patent market, they may help overcome this informational

friction[7].

Generally speaking, it is not clear why the enforcement of patents should per se be considered a bad phenomenon. Litigation has always been part of the patent system as an enforcing mechanism. It is of course costly[8], but it is a necessary evil, since it provides a credible threat to deter imitators from extracting part of the inventor's reward[9]. Whether the compensation for an inventor's research efforts and investments come directly through licensing or litigation on the inventor's part, or through an intermediary entity which conducts these activities on the inventor's behalf, is irrelevant in and of itself–in fact, it matters to the extent that the intermediary may weakly increase the rewards to the inventors, since the latter always have the option of not trading their patents.

Related Literature

In recent years there have been several studies that try to understand the nature of non-practising entities. The literature on NPEs can be divided into two main categories: essays written by law scholars, and papers written by economists, both empirical and theoretical. Most of what is written about NPEs are essays that try to justify why NPEs are either beneficial or detrimental for the economy. A summary of this discussion can be found in Risch (2012) and Lemley and Melamed (2013). Many of these essays take a stand either in favor or against NPEs. Most of the arguments against NPEs are based on their opportunistic behavior as pointed out in Bessen and Meurer (2008). Most of the arguments in favor appeal to the market-maker behavior of NPEs (McDonough (2006); Spulber (2011)).

Empirical papers have focused first on the definition of an NPE. Usually the definition is of an entity that differs from producing firms in terms of what kind of patents it owns and how much it litigates them. Perhaps one of the papers that has attracted most of the attention in the public debate is Bessen and Meurer (2012), where the authors estimate the direct costs that NPEs impose on the patent system. However, their methodology is put into question in Schwartz and Kesan (2012) where they claim that the costs are highly overestimated. Other empirical papers try to estimate what NPEs are doing in the litigation sphere (see for example Shrestha (2010), Pohlmann and Opitz (2013)). One of the most common approaches to tackling the problem in empirical papers is to compare the patent porfolio of a producing entity against that of a nonproducing entity, and draw conclusions about whether NPEs are good or bad based on their differences. For example, Love (2011) studies the age of patents litigated by practicing and non-practicing patentees. Jeruss et al. (2012) shows that litigation by NPEs is increasing and that

---

[7]Of course, many patent trades involving an NPE are conducted in secrecy, and this argument would not apply to those.

[8]As of 2011, according to the American Intellectual Property Law Association, for a claim that could be worth less than a $1 million, median legal costs are $650,000. When the claims range from $1 to $25 millions, the median litigation costs are $2.5 million. And for claims over $25 million, the median legal costs are $5 million. Beside these costs, the median duration of litigation is about 2.5 years.

[9]Consider the case between Apple and Samsung in August 2012. After the two firms did not reach an agreement, Samsung was initially ordered to pay $1.05B USD in damages to Apple.

they settle early in the litigation process (See also Chien (2008)). Risch (2012) shows evidence that some empirical results are reversed, by using a different dataset. Lerner (2006) studies litigation, in particular by NPEs, in the financial sector. Lemley and Melamed (2013) and Scott Morton and Shapiro (2013) review the different strategies to monetize patents used by non-practising entities. One of these strategies, called patent privateering, is also discussed in detail in Golden (2013). Lemley and Shapiro (2006) discusses the royalty stacking problem, which is another strategy used by NPEs, where one firm ends up paying licensing fees to multiple parties, larger than what it *should* pay (although there is on-going debate about what the right value of a patent is).

There are not many papers drawing conclusions about the effect of NPEs on innovation based on a theoretical model. Given the complexity of the problem, there are many second order effects that might be hidden behind complex interactions. There has been some work on the effect of intermediaries on innovation, and some of this literature is reviewed in Howells (2006). Closer to our model is Bessen and Meurer (2006), although its focus is on litigation, rather than the rate of innovation. Perhaps the closest literature to our paper is the one that studies the role of a middleman (Biglaiser (1993); Rust and Hall (2002)). However, the question here is different and we do not assume any special technology for the middleman. Also related to our paper is the literature on patent thickets, which develops after Shapiro (2000). Apart from these papers, as far as we know, there is no theoretical model that provides economic insights on the effect of NPEs on the rate of innovation.

3   Setup

Consider an environment where two firms are involved in researching and developing a new product involving two components. Each one of the components of the product is patentable. Initially, firms know they require the two components to be able to make the final product, but none of the firms has actually developed the technology yet, nor patented anything. Therefore firms must invest in R&D to develop the components. These investments are mapped into discoveries stochastically, which is a common feature of patent races models. When a firm discovers something, a patent is issued freely and immediately. More importantly, we assume that patents are publicly observable[10]. We also assume that once a component has been patented, the firm that does not have a patent on this component could freely imitate it.

We model competition in the final product market in a reduced form: if a single firm sells the good, it makes a profit of π_m, whereas if two firms sell the final product, each makes π_d.[11]

Our model incorporates an imperfect patent system, where it is not possible to determine with certainty

---

[10]This avoids the firm's strategic decision of when to patent, which is an interesting problem on its own.

[11]We can think of π_m and π_d as reduced form representations of a discounted flow of future payoffs, discounted by some depreciation rate, which represents the rate at which the new product becomes obsolete, or the rate at which the market for it shrinks over time.

whether a 'copied' component infringes on the patented one. When a firm patents a new component or technology, its competitor could freely and immediately imitate or invent around it. Because of the imperfection of the patent system, a copied component will not obviously be seen as an infringement on the original inventor's patent. If a competitor were to sell the final product using a copied component, the patent holder could offer a license contract to the imitator. Throughout our analysis we assume that the license contract is one where the patent holder receives a one time fee L from the imitator, for the right to use the component. The imitator may then accept or reject this contract. If the contract has been rejected, the patent owner decides whether to sue for infringement or do nothing.

Crucial to our results are the assumptions about the litigation process: costs, court decisions and penalties. Every time a firm goes to court, it has to pay a fixed legal cost c, which includes lawyers costs, preparation of the case, time in court, etc. This cost is paid by all the parties involved in a trial and the amount c is assumed to be *independent of the number of infringement claims*. This is the key assumption of our model, which is also a common assumption in the patents pool literature. The outcome of a trial is determined by the court and, as we noted above, we focus on an industry where patents have fuzzy boundaries. That is, a court cannot with perfect accuracy recognize whether an imitation infringes on the original patent. We introduce this uncertainty in our model as a random court verdict. With probability $\beta \in (0, 1)$ a copied component infringes on the patented one, and with probability $1 - \beta$ it does not[12]. We assume that all court decisions are independent.

The way the courts award damages to a plaintiff are key to our conclusions. Unfortunately, there is no general and simple formula that can be used to award damages, and in many cases these are contingent on the specific details of a case. However, in most cases there are two elements that contribute to the final compensation to the plaintiff: reasonable royalties and fees per product. Reasonable royalties correspond to forgone royalties for the patent holder, had the infringer bought a license in the first place. These are often the most important part and amount to up to 80% of the final damages. The other component in our model is a per-product fee, which captures any additional damages beyond reasonable royalties awarded to a patent holder–these could include, for example, foregone profits.

We assume the following damages awards. If a producing firm infringes on two patents held by the plaintiff, the damages award is $F + 2R$. Reasonable royalties are paid per component infringed, whereas the 'fixed' component is paid once for any product which has been found to infringe. If the firm infringes on only one component, the award is $F + R$.

Besides the damages award, a patent owner could seek to exclude a rival off the market through an

---

[12]More generally, we can interpret β as a belief about how strong the patent portfolio of the firm is. In reality firms often own large portfolios of patents, so there might be other patents w, z that can be used in court against x or y. Since firms may be unaware of the complete portfolio of their rivals, or it may be very costly to know exactly what each one of the claims is protecting, we can interpret β as the probability of winning the lawsuit, accounting for the court's inherently random decision, plus uncertainty regarding the complete portfolio of the rivals. Finally, we assume that the outcomes of different patent infringement lawsuits are independent events.

injunction. Although not granted very often, injunctions do play a role in the enforcement of IP when the patent holder is a producing firm. Historically, NPEs used to be treated similarly until the 2006 eBay vs Motorola decision, following which NPEs are no longer awarded injunctions. Today the only way for an NPE to get an injunction is generally through the ITC, which is also not very common. Therefore in our model we rule out the possibility of injunctions for NPEs.[13] On the other hand, in lawsuits involving two producing firms the plaintiff may be awarded an injunction against its competitor, with probability I.

The final ingredient of our model is the timing of the actions. First is the research stage where firms invest in order to make discoveries. Once all discoveries are made and the patent portfolios are determined, trade between firms and the NPE takes place in the form of take it or leave it offers by the patent owner[14]. Our model assumes *no commitment* on the firm side: if a firm sold its entire patent portfolio to a competitor, the firm cannot commit to stay out of the market. Thus even if one firm acquired the patents for both components, the competitor could still enter the market and risk infringement. This feature allows us to have a model where the solution is not the trivial one: one firm buys up all the patents and produces as a monopolist[15]. Following any patent trades, firms decide whether to enter the final product market, patent owners offer licences to entrants, and any potential litigation takes place following licence rejections.

Assumptions

We operate under five assumptions that guarantee our results.[16] These conditions—dealing with the nature of competition, litigation, licensing and entry—guarantee: 1) that industry profits would be maximized by a monopoly regime; 2) litigation is profitable when a firm or an NPE's license offer is rejected; 3) firms and the NPE prefer to extract surplus through licensing rather than litigation; 4) entry in the final product market is profitable. Besides that, for simplicity we assume that the damage award in a lawsuit is the same for firms and the NPE,[17] though we have shown that analogous results hold when we allow damages to differ systematically.

Our first assumption regards the structure of the final product market. We assume that a monopoly earns larger profits than total industry profits under competition. We assume that whatever profits the industry can sustain as a duopoly should also be attainable by a monopolist, who can exactly replicate the actions of each firm in a duopoly (including, for example, by producing multiple differentiated products).

Assumption 1. (Market structure) $\pi_m \geq 2\pi_d$.

The second assumption is over the gains from litigation. Since litigation is costly it will be a credible

---

[13]This is not important for the overall conclusion of this paper. An earlier version of the paper treats producing and non-producing firms identically, including injunctions for both, and yields analogous conclusions.

[14]This TIOLI assumption is not crucial and we elaborate on this point in the Appendix.

[15]There may also be other reasons why this would not be a satisfying assumption, e.g. anti-trust concerns.

[16]We have a more detailed solution of the game for a general set of parameters, but here we restrict to this set of assumptions for the sake of exposition. The full analysis is available upon request.

[17]I.e. a successful plaintiff would receive $F + k \cdot R$, regardless of whether it is a producing firm or not, for a fixed k.

threat only when its expected payoff is positive.

Assumption 2. (Litigation) $c \leq \beta(F + R)$.

Next, we assume that, whenever possible, firms would prefer to agree on licenses rather than go to costly litigation. We want litigation to be a credible threat that does not occur on the equilibrium path. In order for this to happen, we impose a condition that compares the costs and gains from litigation.

Assumption 3. (Licensing) $c \geq \frac{\beta(2-\beta)}{2}I(\pi_m - 2\pi_d)$.

We impose a condition on entry. If a market can sustain only one firm there is nothing interesting to analyze. Therefore we assume that duopoly profits are high enough so entry is attractive for both firms.

Assumption 4. (Entry) $\pi_d \geq \frac{2\beta(F+R)+2c}{1-\beta I}$.

Under these conditions we study the effect of an NPE on the equilibrium payoffs. For the sake of exposition we restrict attention to the case where the NPE has all the bargaining power. However, this is not crucial to the results and Appendix C generalizes the results when the NPE has only partial bargain power. Finally, we assume one last condition, which guarantees that a firm which owns both patents is willing to sell one of them to the NPE:

**Assumption 5.** (Sell-1) $c + \beta^2 F \geq \beta(1 - \beta)I\pi_d$.

Remark: There is a non-empty set of parameters for which these conditions simultaneously hold. This is easy to see when $I \to 0$.

Description of the Research Stage

We borrow from the models of multi-stage innovation to describe the research process. Firms will invest to discover the two components required to sell the product. We do not assume that research investments have to be sequential, since in principle the two components are two unrelated pieces of technology. Thus, we do not require firms to finish one stage before starting the next one. Initially, firms decide how much effort to exert on the discovery of each component. Effort is costly and firms share the same cost function $c(e)$ which is increasing and convex. Depending on the firm's effort, success arrives according to an exponential random variable. If at $t = 0$ firm i allocates an amount of effort x_i to discover one component, the unconditional probability of an arrival before time τ is given by $1 - \exp(-x_i\tau)$.

Since there is no learning and the exponential distribution is memoryless, we focus on a Markov perfect equilibrium, where the state is given by the history of discoveries. Each firm will revise their effort decisions only when there is a new discovery, which is publicly known.

Figure 1. Research Stage: How initial patent portfolios are determined.
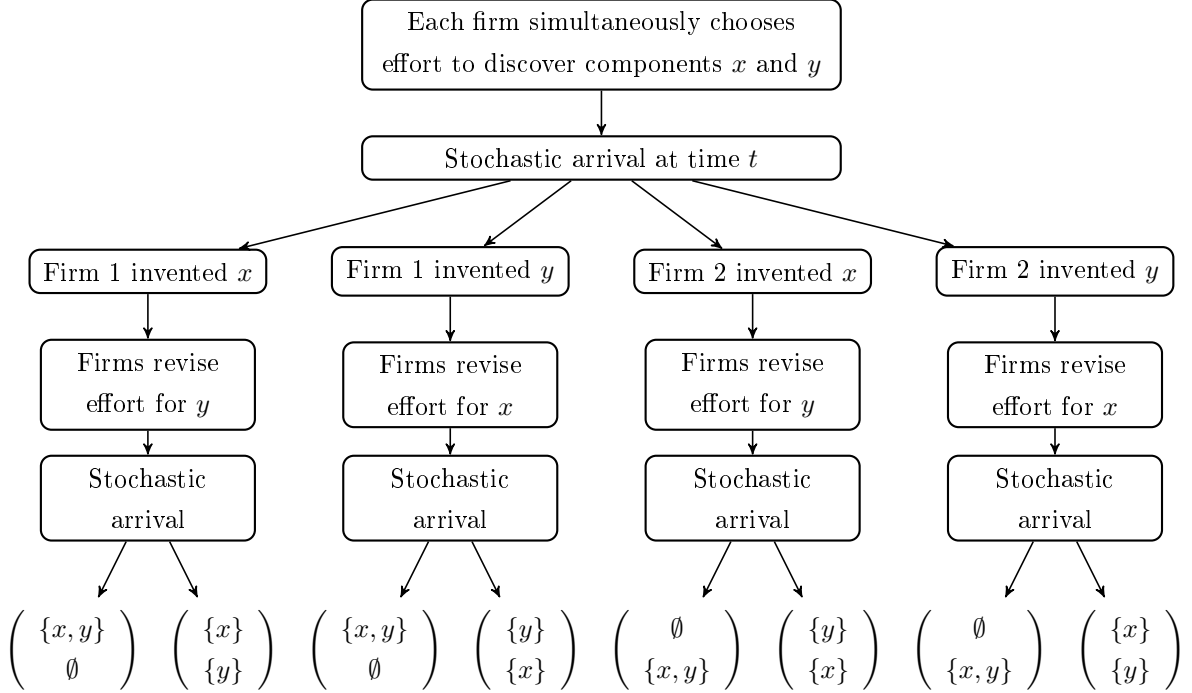
Since firms and components are modeled symmetrically, there are essentially only two possible patent landscapes: 1) when one firm owns both patents, i.e has the patent portfolio $\{x, y\}$, and the rival has an empty portfolio $\emptyset$; 2) when each firm one patent (for example when firm 1's portfolio is $\{x\}$ and firm 2's portfolio is $\{y\}$).

In the following sections we analyze the subsequent stages: patent acquisition, litigation, licensing agreements and court outcomes in order to determine the value of a given patent portfolio. Once we determine the value of each patent portfolio, we can use it as an input in the research stage to determine the effort exerted by each firm.

We analyze two cases. In the first case there are no NPEs in the economy. That is, there are no firms acting as an intermediary, buying patents and selling licenses under the threat of litigation. In the second case, we introduce an NPE and we allow patent transactions between producing firms and the NPE.

4   An economy without NPEs

Suppose firms can only trade patents amongst themselves after the discoveries are made, and in the continuation game they sell licenses or litigate according to strategies that are subgame perfect. Because firms are 'anonymous' in our model, it is enough to focus on two subgames:

Game 1:   Firm 1 owns both patents, i.e has the patent portfolio $\{x, y\}$, and firm 2 has an empty portfolio $\emptyset$.

Game 2: Each firm owns one patent: firm 1's portfolio is $\{x\}$ and firm 2's portfolio is $\{y\}$.

We use the subgame perfect equilibrium concept to calculate the payoffs from these two games. In what follows we lay out the structure of each subgame and study the equilibrium under the assumptions that we stated above. The full analysis of each subgame, for any parameter values, is available, but not included here for the sake of exposition.

Game 1

This subgame starts after firm 1 has just become the owner of both patents. Firm 2 first decides to enter or not. If it does not enter, firm 1 becomes a monopolist. If firm 2 enters, firm 1 offers a license contract for each one of the components. Firm 2 decides to accept the contract for one component, two components or none. After a rejection, firm 1 makes the decision of going to court or not.

Figure 2. Continuation Game 1: Patent portfolios are $\{x, y\}$ for firm 1 and $\emptyset$ for firm 2.
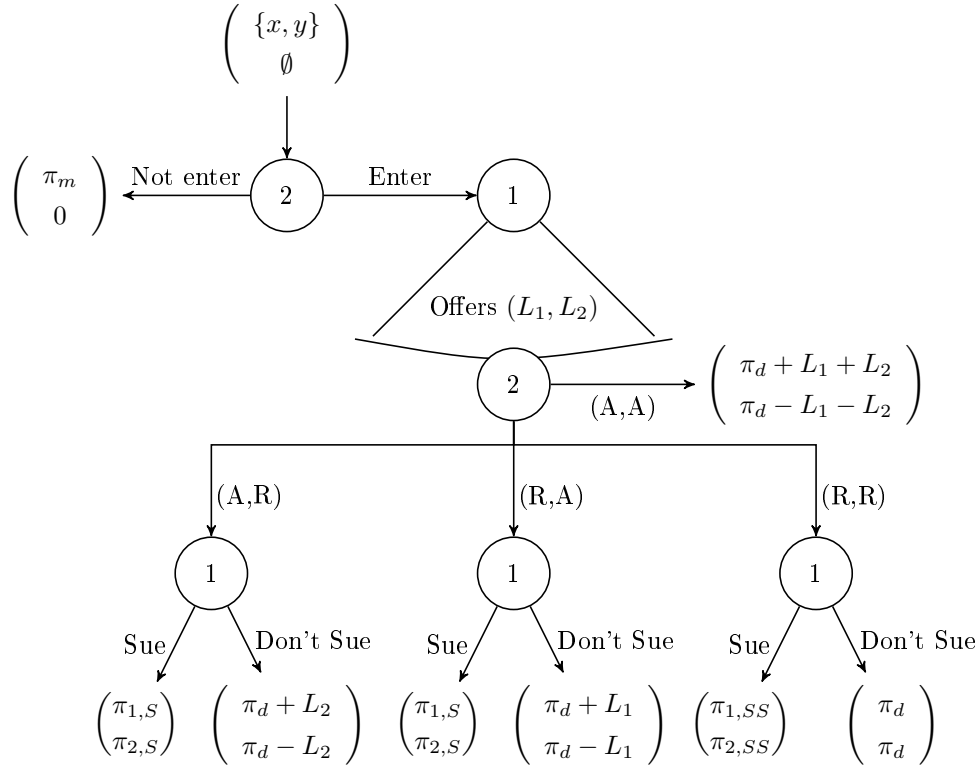
Where $\pi_{i,S}$ is the firm i's payoff after a one component lawsuit and $\pi_{i,SS}$ is firm i's payoff after a two component lawsuit.

Analysis of Game 1

We solve the subgame by backward induction. Suppose firm 2 rejects both contracts and firm 1 decides to sue on both components. Our assumptions on damages is that firm 1 gets F if some component is

infringed by the rival's product, plus R in reasonable royalties for each infringed component. In addition, with probability I an injunction is granted against the infringer and, as a consequence, firm 2 has to leave the market. The cost of a trial is c, independent of the number of patents brought in the law suit. Therefore the payoffs of a two component law suit are given by:

$$\pi_{1,SS} = \pi_d - c + 2\beta R + \beta(2-\beta)[F + I(\pi_m - \pi_d)], \ \pi_{2,SS} = \pi_d - c - 2\beta R - \beta(2-\beta)[F + I\pi_d].$$

When firm 2 rejected both licenses there are three possible suing strategies that firm 1 could implement: Suing on both components in the same trial, suing in two separate simultaneous trials, or suing both component in sequential trials.

Suing for both components in the same trial avoids the royalty stacking problem, since the fixed component of the damages award F will be paid only once. Therefore, the payoff for firm 1 is

$$\pi_{1,SS} = \pi_d - c + 2\beta R + \beta(2-\beta)[F + I(\pi_m - \pi_d)]$$

But if firm 1 files two simultaneous independent law suits, it doubles its potential fixed payment F, as well as the cost c. In this case, the payoff for firm 1 is:

$$\pi_{1,Sue1} = \pi_d - 2c + 2\beta(F + R) + \beta(2-\beta)I(\pi_m - \pi_d)$$

Finally, the other posibility is to sue once for infringement on one component, and after learning the outcome of that trial, decide whether to sue again (which will only happen if firm 2 has not been excluded through an injunction). In this case firm 1's payoff is

$$\pi_{1,Seq} = \pi_d - c + \beta(F + R + I(\pi_m - \pi_d)) + (\beta(1-I) + (1-\beta))[\beta[F + R + I(\pi_m - \pi_d)] - c].$$

It is easy to see that $\pi_{1,SS} \geq \pi_{1,Sue1} \Leftrightarrow c \geq \beta^2 F$ and it's always true that $\pi_{1,SS} \geq \pi_{1,Seq}$.[18] Our main result does not depend crucially on this strategic decision, and we will restrict attention to the case where firm 1 sues for both components in one trial, i.e. $c \geq \beta^2 F$.

Consider the case where firm 2 accepted one license and infringed on the other component, and firm 1 decided to sue on that component. In this case, the payoffs are given by:

$$\pi_{1,S}^{L_i} = \pi_d - c + L_i + \beta[F + R + I(\pi_m - \pi_d)], \ \pi_{2,S}^{L_i} = \pi_d - c - L_i - \beta[F + R + I\pi_d].$$

Firm 1 prefers to sue on both components rather than none iff $\pi_{1,SS} \geq \pi_d$, equivalent to

$$c \leq 2\beta R + \beta(2-\beta)[F + I(\pi_m - \pi_d)].$$

If firm 2 accepted only one license, firm 1 prefers to sue on the remaining component rather than not iff $\pi_{1,S}^{L_i} \geq \pi_d + L_i$ which is equivalent to

$$c \leq \beta R + \beta[F + I(\pi_m - \pi_d)].$$

---

[18] Anecdotical evidence suggests that there is an increasing amount of communication among courts and they may deny double awards for lawsuits filed in different courts.

Assumption 2 guarantees that these two conditions above hold. Thus firm 1 will always sue after firm 2 rejected any the contract.

Consider the contract (L_1, L_2), knowing that firm 1 will sue firm 2 if a contract is rejected. We define the following auxilliary variables:

$$\alpha = c + \beta[F + I\pi_d + R], \ \hat{\alpha} = c + \beta(2 - \beta)[F + I\pi_d] + 2\beta R, \ \gamma = \hat{\alpha} - \alpha = \beta R + \beta(1 - \beta)[F + I\pi_d].$$

Figure 3. Continuation strategy of firm 2, for a given license contract.
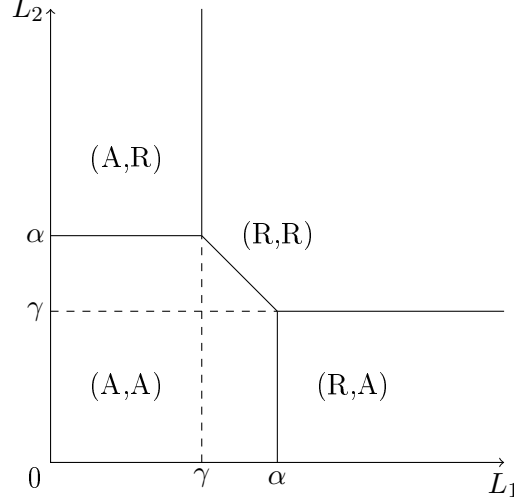
Accepts both if	$\max\{L_1, L_2\} < \alpha$ and $L_1 + L_2 < \hat{\alpha}$
Accepts 1 and rejects 2 if	$L_1 < \gamma$, $L_1 < L_2$ and $\alpha < L_2$
Accepts 2 and rejects 1 if	$L_2 < \gamma$, $L_2 < L_1$ and $\alpha < L_1$
Rejects both if	$L_1 + L_2 > \hat{\alpha}$ and $\min\{L_1, L_2\} > \alpha$.

By offering a contract firm 1 is choosing the payoff of the subsequent stage.

If firm 1 wants firm 2 to accept both contracts, the best it can do is to offer (L_1^*, L_2^*) such that $\max\{L_1^*, L_2^*\} \leq \alpha$ and $L_1^* + L_2^* = \hat{\alpha}$. If firm 1 wants firm 2 to accept contract i and reject the other one, the best it can do is to offer $L_i^* = \gamma$ and $L_{-i}^* > \alpha$. If firm 1 wants firm 2 to reject both contracts, the best it can do is to offer (L_1^*, L_2^*) such that $\min\{L_1*, L_2^*\} > \gamma$ and $L_1^* + L_2^* > \hat{\alpha}$.

Notice that firm 1 always prefers firm 2 to reject both contracts rather than accept only one. The most firm 1 can extract from firm 2 by making firm 2 accept one contract and reject the other one is γ.

$$\pi_d - c + \gamma + \beta[F + R + I(\pi_m - \pi_d)] \geq \pi_d - c + 2\beta R + \beta(2 - \beta)[F + I(\pi_m - \pi_d)] \Leftrightarrow 2\pi_d \geq \pi_m.$$

The intuition behind this result is the economies of scale in law suits plus the injunction possibility. The infringer pays the same cost c when sued on two components or only one. Therefore, the license that firm 1 can extract from firm 2 does not compensate the gain of suing on both components: it increases the probability of injunction and does not raise the litigation cost.

Next, firm 1 prefers that firm 2 accept both contracts instead of rejecting them iff

$$\pi_d + \hat{\alpha} \geq \pi_{1,SS} \Leftrightarrow c \geq \frac{\beta(2-\beta)I(\pi_m - 2\pi_d)}{2}.$$

These conditions are guaranteed by Assumption 3. Given that licenses are chosen optimally for firm 1, they make firm 2 indifferent between accepting or rejecting both contracts. Firm 2 will decide to enter if and only if it makes non negative profits. We have the following entry condition:

$$\pi_d \geq \frac{\beta(2-\beta)F + c + 2\beta R}{1 - \beta(2-\beta)I}.$$

Assumption 4 guarantees that entry is profitable. Therefore, the equilibrium is: Firm 2 enters and firm 1 offers contracts that are accepted. Payoffs are $\pi_1 = \pi_d + \hat{\alpha}$, $\pi_2 = \pi_d - \hat{\alpha}$.
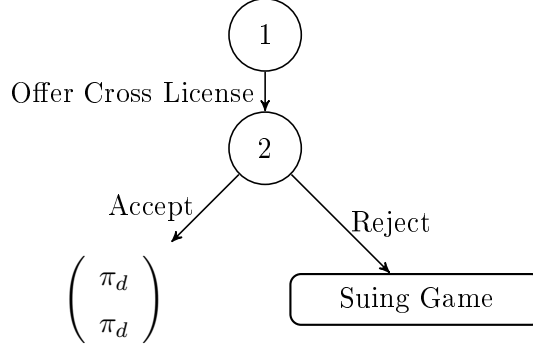
Game 2

This is the subgame where the two components have been discovered by different firms, and each firm holds one patent. Without loss of generality, we assume firm 1 owns the patent for x, while firm 2 owns that for y. Firms simultaneously decide whether to enter the final product market or not. Both firms could decide to infringe on each other and participate in the market at the same time. On the other hand, they could agree on cross-licensing and avoid litigation, in which case each firm gets π_d. The other possibility is that they go to litigation.

Figure 4. Continuation Game 2
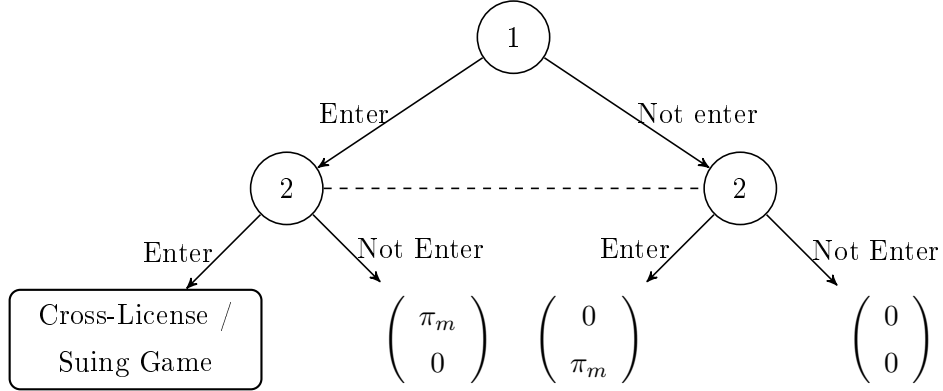
Since firms are symmetric, which firm offers cross licensing and who accepts or rejects this offer is irrelevant. Assume the Cross-Licesing/Suing game is as follows:

Figure 5. Cross-Licensing / Suing Game

If firms do not agree on a cross-license they simultaneously choose whether or not to sue each other.

Analysis of Game 2

We solve this game by backward induction. First, consider the simultaneous litigation game, following a history where both firms have entered the final product market:

Figure 6. Simultaneous litigation game.

	Sue	Not Sue
Sue	$(\pi_{S,S}, \pi_{S,S})$	$(\pi_{S,NS}, \pi_{NS,S})$
Not Sue	$(\pi_{NS,S}, \pi_{S,NS})$	$(\pi_d, \ \pi_d)$

If both firms sue each other they have an expected payoff of: $\pi_{S,S} = \pi_d - 2c + \beta(1-\beta)I(\pi_m - 2\pi_d)$. If one firm sues and the other firm does not, the expected payoffs are, respectively:

$$\pi_{S,NS} = \pi_d - c + \beta[F + R + I(\pi_m - \pi_d)], \quad \text{and} \quad \pi_{NS,S} = \pi_d - c - \beta[F + R + I\pi_d].$$

Assumption 3 guarantees that $\pi_{S,S} < \pi_d$ and Assumption 2 guarantees that $\pi_{S,S} > \pi_{NS,S}$ which implies that (S, S) is the unique equilibrium in the litigation game[19]. Therefore, firms never want to reach the litigation stage and the unique equilibrium is that firms will cross-license and the payoffs are π_d for both firms.

5 NPE as a third player

In this section we study the same model of R&D and competition, but we introduce an NPE as a new player. As pointed out in the introduction the NPE will only differ from a firm in that it has no capacity to do R&D or to produce a product. Initially, it does not own patents and it can only resort to patent trade to make profits. After the firms make the discoveries, the NPE may offer a contract to buy a patent, including a grant-back clause. This contract establishes a buying price p, plus a **non-exclusive** licensing

---

[19]Another way to rule out the litigation outcome is to assume that countersuing is free (or much cheaper than c)

deal with firm i. In the following four sections we analyze all possible licensing and litigation subgames, depending on which entity owns which patent (we call these subgames A,B, C and D).
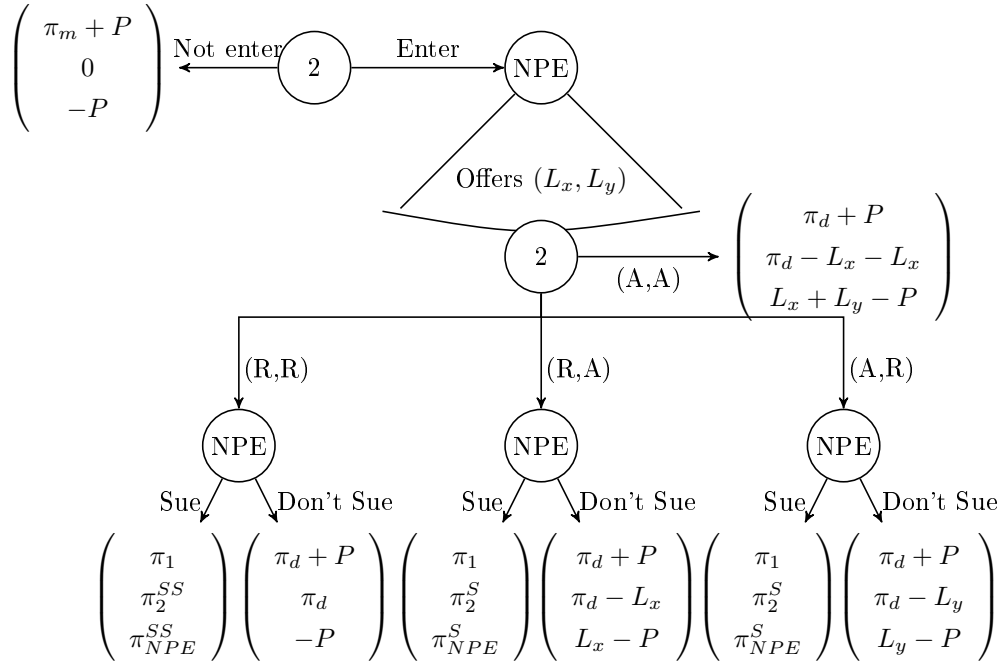
Game A

Consider the game where the NPE owns both patents, and it acquired both of them from firm 1. The patent portfolios are then: NPE=$\{x, y\}$, firm 1=$\emptyset$, firm 2=$\emptyset$, while patent protection is: firm 1=$\{x, y\}$, firm 2=$\emptyset$. Patent protection refers to the grant-back clauses that we assume are included in a trade involving an NPE.

Firm 1 will always enter the market, since in the worse case it gets $\pi_d + p_1 + p_2$. Firm 2 has to decide to enter or not. If it chooses to enter, the NPE will offer it a menu of licenses, (L_x, L_y), one for each patent. If any license is rejected, the NPE has the option to sue firm 2 for infringement.

Figure 7. Continuation Game A:

The NPE owns both patents and acquired them from firm 1. Denote $P = p_1 + p_2$.

Analysis of Game A

This game is similar to Game 1, because the NPE makes all the suing decisions just like firm 1 did in Game 1. Our assumptions on damages are that the NPE gets F if some component is infringed by firm 2's product, plus R in reasonable royalties for each infringed component. The main strategic difference

between firms and the NPE is that the NPE *does not receive injunctions*[20]. As before, the cost of a trial is c, independent of the number of patents brought in the law suit.

Define $\tilde{\alpha} = c + \beta[F + R]$, $\tilde{\hat{\alpha}} = c + \beta[(2 - \beta)F + 2R]$, $\tilde{\gamma} = \tilde{\hat{\alpha}} - \tilde{\alpha} = \beta[(1 - \beta)F + R]$, which are analogous to the parameters in Game 1.

First, notice that if firm 2 refused a licence for either component, the NPE would sue iff $c \leq \beta[(2 - \beta)F + 2R]$, whereas if firm 2 refused exactly 1 licence, the NPE would sue iff $c \leq \beta[F + R]$. Assumption 2 guarantees that these two conditions hold, so the NPE's litigation threats are credible in either case. Thus every firm's payoffs, depending on whether firm 2 rejected 2, 1, or 0 licences are, respectively:

$$\pi_1 = \pi_d, \ \pi_2 = \pi_d - \tilde{\hat{\alpha}}, \ \pi_{NPE} = \tilde{\hat{\alpha}} - 2c$$
$$\pi_1 = \pi_d, \ \pi_2 = \pi_d - \tilde{\alpha} - L_i, \ \pi_{NPE} = \tilde{\alpha} - 2c + L_i$$
$$\pi_1 = \pi_d, \ \pi_2 = \pi_d - L_1 - L_2, \ \pi_{NPE} = L_1 + L_2$$

Given license contracts (L_1, L_2), the regions of acceptance and rejection for firm 2 are analogous to the ones in figure 3, in the analysis of game 1, except that we need to replace α by $\tilde{\alpha}$, γ by $\tilde{\gamma}$ and $\hat{\alpha}$ by $\tilde{\hat{\alpha}}$.

Thus, as in game 1, the NPE can choose the payoffs by deciding the license contract. Notice, however, a difference between firms and the NPE. Since the NPE does not get injunctions, it is indifferent between getting one or two license contracts accepted. The most the NPE can extract in licensing both component is $\tilde{\hat{\alpha}}$, and the NPE prefers the two licenses to be accepted rather than to sue on both of them iff $\tilde{\hat{\alpha}} = c + \beta[(2 - \beta)F + 2R] \geq \tilde{\hat{\alpha}} - 2c$, which always holds.

Firm 2 will enter only if the entry condition $\pi_d \geq c + \beta(2 - \beta)F + 2\beta R$ holds. Assumption 4 guarantees this is the case. Thus, the equilibrium of the subgame is: Firm 2 enters, the NPE offer licenses that are accepted and the payoffs are $\pi_1 = \pi_d + p_1 + p_2$, $\pi_2 = \pi_d - \tilde{\hat{\alpha}}$, $\pi_{NPE} = \tilde{\hat{\alpha}} - p_1 - p_2$.

Game B

Consider the game where the NPE owns both patents, and it acquired one from firm 1 and one from firm 2. The patent portfolios are then: NPE=$\{x, y\}$, firm 1=$\emptyset$, firm 2=$\emptyset$, while patent protection is: firm 1=$\{x\}$, firm 2=$\{y\}$.

Firms 1 and 2 first simultaneously decide whether to enter or not.

---

[20]We have also explored the case where a producing firm can be excluded with an injunction when the patent holder is an NPE. The main result of our paper is unchanged.

Figure 8. Continuation Game B
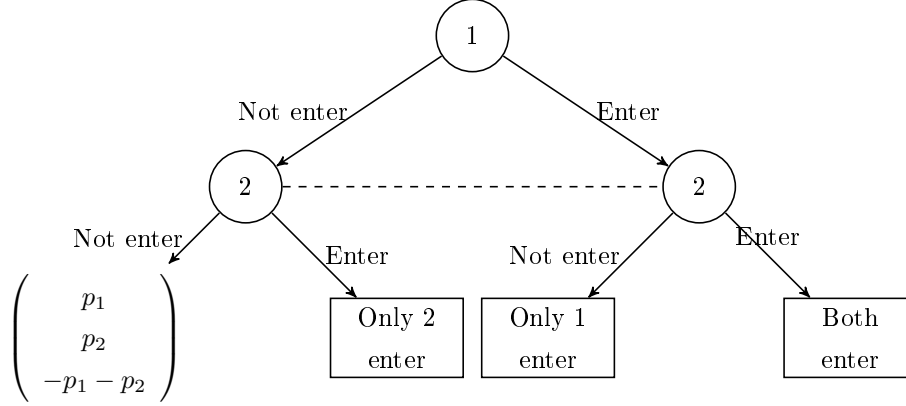
Consider the subgame where both firms have entered. The NPE is going to make offers L_y to firm 1 and L_x to firm 2. The firms then simultaneously choose whether to get a license from the NPE or not. If they decide not to, the NPE decides whether to sue the infringers or not.

Figure 9. Both firms enter
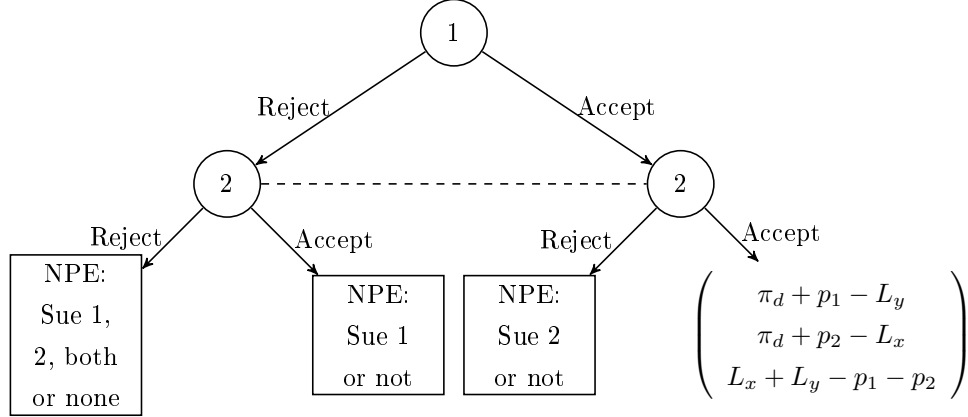
Analysis of Game B

If a firm rejects the NPE's licence offer, the NPE would sue if and only if $c \leq \beta(F + R)$ (Assumption 2), regardless of whether the other firm accepted or rejected a licence.

Suppose both firms entered. Notice that each firm's decision to accept or reject the NPE's licence offer is independent of whether the other firm accepts or rejects the offer it is given by the NPE. In particular, firm i's payoff from accepting the NPE's licence at price L is $\pi_d - L$, while its payoff from rejecting it is $\pi_d - \beta(F + R) - c$. Therefore, the NPE's optimal licences are such that each firm is made indifferent between accepting and rejecting it. This is $L^* = \beta(F + R) + c$. The only way for the NPE to earn profits is by selling licenses. Therefore, the equilibrium payoffs are:

$$\pi_i = \pi_d - \beta(F + R) - c + p_i, \quad \pi_{NPE} = 2\beta(F + R) + 2c - p_1 - p_2.$$

Suppose only one firm entered. As in the previuos case the NPE offers a license at price $L^* = \beta(F+R)+c$

and its accepted. Suppose firm i entered and firm j stayed out. The equilibrium payoffs are:

$$\pi_i = \pi_m - \beta(F + R) - c + p_i, \ \pi_j = p_j \text{ and } \pi_{NPE} = \beta(F + R) + c - p_1 - p_2.$$

If no firm entered, the equilibrium payoffs are $\pi_1 = p_1, \ \pi_2 = p_2$ and $\pi_{NPE} = -p_1 - p_2$.

Next, consider the firms' entry decisions. Assumption 4 guarantees that $\pi_d \geq c + \beta(F + R)$ and therefore, in equilibrium: Both firms enter, the NPE offers licenses $L^* = \beta(F + R) + c$ to each firm and they accept. The equilibrium payoffs are: $\pi_i = \pi_d - \beta(F + R) - c + p_i$ and $\pi_{NPE} = 2(\beta(F + R) + c) - p_1 - p_2$.

Game C

Consider the game where the NPE owns the patent for x and it was acquired from firm 1, while firm 2 owns the patent for y. The patent portfolios are then: NPE=$\{x\}$, firm 1=$\emptyset$, firm 2=$\{y\}$, while patent protection is: firm 1=$\{x\}$, firm 2=$\{y\}$.
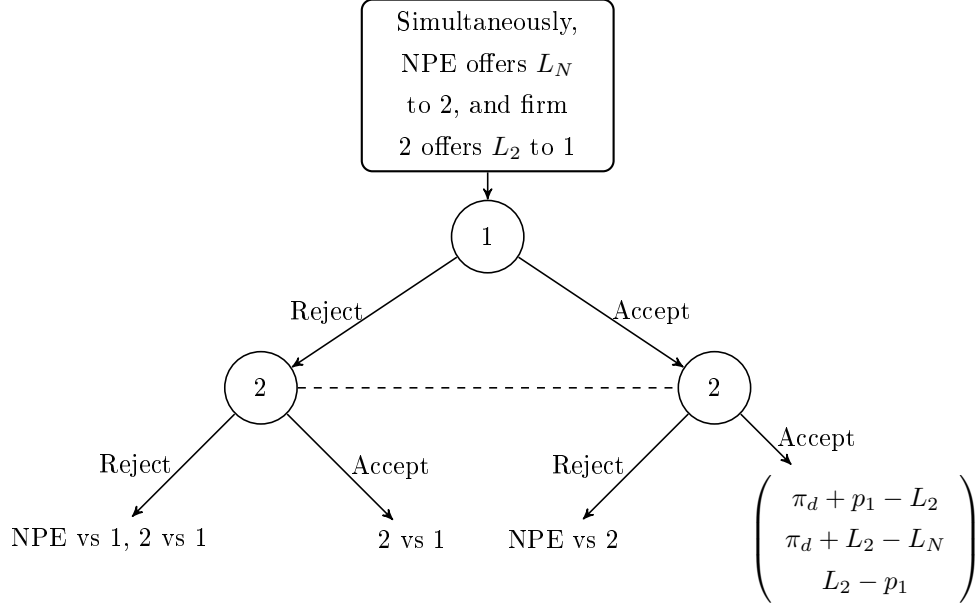
Firms simultaneously decide whether to enter or not, then license offers are made simultaneously, and finally potential litigation decisions are made simultaneously.

Figure 10. Continuation Game C
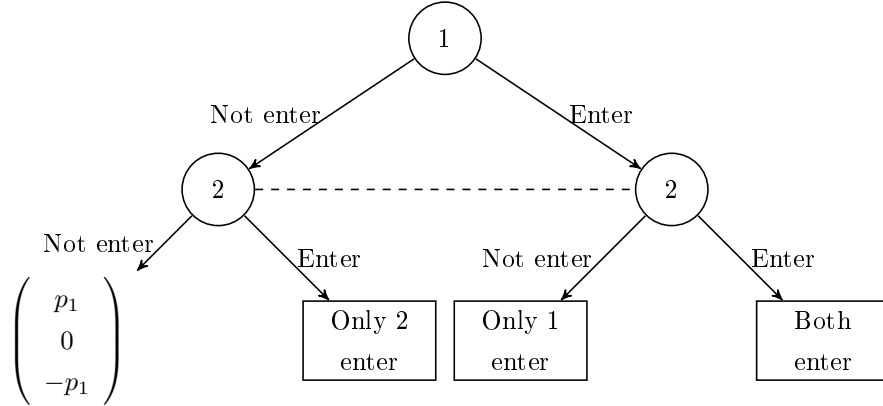
The subgame where both firms entered is depicted below.

Figure 11. Both firms entered

If only firm 1 enters, then firm 2 makes a licensing offer under the threat of litigation. If only firm 2 entered, the NPE makes the licensing offer.

Analysis of Game C

The lawsuits by firm 2 and the NPE are strategically independent of each other. Assumption 2 implies $\beta(F+R) > c$, so litigation is credible for the NPE, and $\beta(F+R)+\beta I(\pi_m - \pi_d) > c$, so it is also credible for firm 2.

Suppose both firms entered. The highest licenses that will be accepted are $L_N^* = \beta(F+R)+c$ for the NPE and $L_2^* = \beta(F+R+I\pi_d)+c$ for firm 2. The NPE will always want its license to be accepted. However firm 2 might prefer to litigate in order to exclude firm 1 off the market through an injunction. Firm 2 prefers to offer a licence of L_2^* to firm 1 if and only if

$$c > \frac{\beta I(\pi_m - 2\pi_d)}{2}.$$

This condition holds by Assumption 3.

Suppose only firm 1 entered. Even when firm 1 infringes on firm 2's patent, since firm 2 is not producing, there will be no injunction. Therefore, the maximum firm 2 can offer is $L_2^* - \beta I\pi_d$ to firm 1. In this case, firm 2's license will always be offered and accepted.

Suppose only firm 2 entered. The NPE offers a licence L_N^* to firm 2 and it will be accepted.

Therefore, if both firms entered, firm 2 and the NPE sell licenses that are accepted and the equilibrium payoffs are: $\pi_1 = \pi_d - L_2^* + p_1$, $\pi_2 = \pi_d + L_2^* - L_N^*$, $\pi_{NPE} = L_N^* - p_1$.

If only firm 1 entered: $\pi_1 = \pi_m - \beta(F+R) - c + p_1$, $\pi_2 = \beta(F+R) + c$, $\pi_{NPE} = -p_1$.

If only firm 2 entered: $\pi_1 = p_1$, $\pi_2 = \pi_m - \beta(F+R) - c$, $\pi_{NPE} = \beta(F+R) + c - p_1$.

Finally, consider the entry stage. Let $\pi_i^{E,E}$ be the payoff of firm i in the continuation game where both firms entered.

Figure 12. Simultaneous entry game

|	Enter	Not enter
Enter	$\pi_1^{E,E}, \pi_2^{E,E}$	$\pi_m - \beta(F+R) - c, \beta(F+R) + c$
Not enter	$0, \pi_m - \beta(F+R) - c$	$0, 0$

$\pi_1^{E,E} = \pi_d - c - L_2^*$, $\pi_2^{E,E} = \pi_d + L_2^* - L_N^* = \pi_d + \beta I \pi_d$. Therefore, entry is a dominant strategy for firm 1 if $\pi_d \geq \dfrac{c + \beta(F+R)}{1 - \beta I}$ and for firm 2 if $\pi_d \geq \dfrac{c + \beta(F+R)}{1 + \beta I}$. Notice that Assumption 4 implies both conditions. Therefore, the unique equilibrium is both firms enter and license offers are accepted, so payoffs are $\pi_1 = \pi_d - L_2^* + p_1$, $\pi_2 = \pi_d + L_2^* - L_N^*$, $\pi_{NPE} = L_N^* - p_1$.

Game D

Consider the game where the NPE owns the patent for x and it was acquired from firm 1, while firm 1 owns the patent for the second component y. The patent portfolios are then: NPE=$\{x\}$, firm 1=$\{y\}$, firm 2=$\emptyset$, while patent protection is: firm 1=$\{x, y\}$, firm 2=$\emptyset$.

Firm 1 will decide to enter, because the lowest payoff it can guarantee itself is $\pi_d + p_1$. Firm 2 decides whether enter, and if it does, firm 1 and the NPE make simultaneous offers for the patents. If offers are rejected, each entity can decide whether to sue for infringement or not. The key difference between a lawsuit from firm 1 versus one from the NPE is the injunction assumption.

Figure 13. Continuation Game D
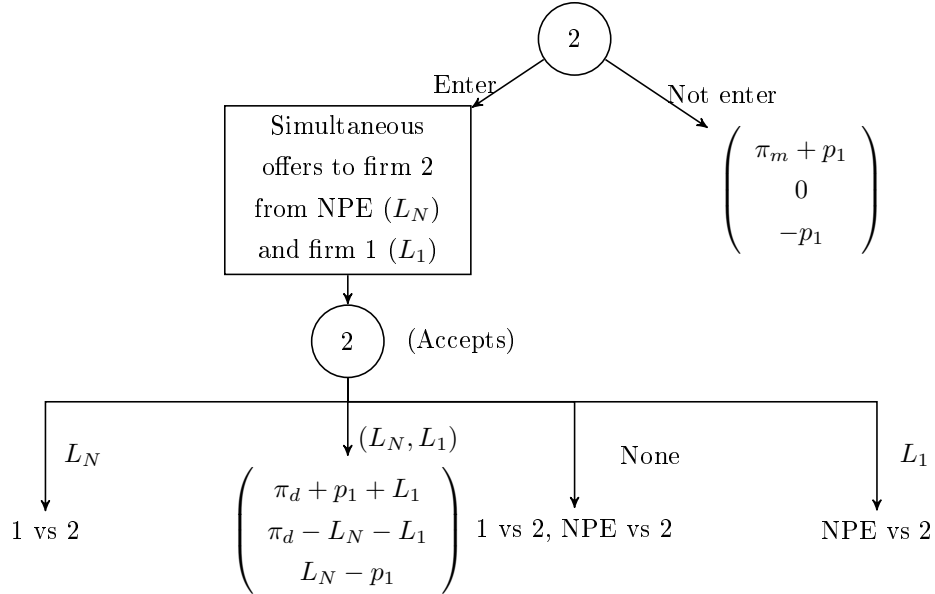
Analysis of Game D

The payoffs under our assumptions are:[21]

- If firm 2 rejects both L_N and L_1:
  $\pi_1 = \pi_d - c + \beta[F+R+I(\pi_m-\pi_d)]$, $\pi_2 = \pi_d - \beta[F+R+I\pi_d] - \beta(F+R) - 2c$, $\pi_{NPE} = \beta(F+R) - c$.
- If firm 2 accepts L_N and rejects L_1:
  $\pi_1 = \pi_d - c + \beta[F+R+I(\pi_m-\pi_d)]$, $\pi_2 = \pi_d - c - \beta[F+R+I\pi_d] - L_N$, $\pi_{NPE} = L_N$.
- If firm 2 rejects L_N and accepts L_1:
  $\pi_1 = \pi_d + L_1$, $\pi_2 = \pi_d - c - L_1 - \beta(F+R)$, $\pi_{NPE} = \beta(F+R) - c$.
- If firm 2 accepts both L_N and L_1:
  $\pi_1 = \pi_d + L_1$, $\pi_2 = \pi_d - L_1 - L_N$, $\pi_{NPE} = L_N$.

Notice that because the NPE is never awarded with injunctions, the suing decision of the NPE does not affect the market structure. Therefore, the suing decision by firm 1 and the NPE are strategically independent. No matter what strategy the NPE plays, firm 1 will sue firm 2 on one component after the rejection of a license contract if and only if $c \leq \beta[F+R+I(\pi_m-\pi_d)]$. The NPE will sue firm 2 after the rejection of a license contract iff $c \leq \beta(F+R)$. Assumption 2 guarantees these conditions hold. Here we can see that the incentives to sue for the firm and the NPE are different. In particular, the NPE has always **less** incentives to sue than firm 1. This is, for $\beta(F+R) < c \leq \beta[F+R+I(\pi_m-\pi_d)]$ firm 1 will sue after a rejection but the NPE will not. Thus, the injunction allows firm 1 to charge more. Under Assumption 2, the firm and the NPE will sue after a rejection.

For given licenses (L_1, L_N) firm 2's optimal choices are: Accept the license from 1 iff $L_1 \leq \alpha$ and accept the license from the NPE iff $L_N \leq \tilde{\alpha}$.

Notice the NPE always want its license to be accepted, because it gets $\tilde{\alpha}$ and going to trial gives $\tilde{\alpha} - 2c$. Firm 1, however, would want to litigate under some conditions. This is because through litigation firm 1 can obtain a market advantage via an injunction decision. Firm 1 would prefer to sell a license and obtain α rather than litigate iff

$$c \geq \frac{1}{2}\beta I(\pi_m - 2\pi_d).$$

Assumption 3 guarantees this condition holds. This equation expresses the trade off between going to trial and selling a license. It can be written as:

$$\beta I(\pi_m - \pi_d) \leq c + [c + \beta I\pi_d].$$

The term $c + \beta I\pi_d$ represents the amount firm 1 can extract from firm 2 from the threat of litigation. Since we assume TIOLI offers, firm 2 is willing to pay to avoid the litigation costs and the risk of infringement. The term $\beta I(\pi_m - \pi_d)$ correponds to firm 1's injunction benefit. Therefore, firm 1 will rather get licenses

---

[21]Suppressing the p_1, p_2 payments for this part of the analysis, since at this point they are sunk.

if the benefit from litigation is smaller than the direct cost of litigation c plus the opportunity cost of selling a license $c + \beta I \pi_d$.

Finally, firm 2 decides to enter or not. Firm 2 will enter if and only if $\pi_d > \alpha + \tilde{\alpha}$, which is implied by Assumption 4.

6   Results

6.1   Patent Trade

Under Assumptions 1 to 5, we have the equilibrium strategies and payoffs in each subgame.[22] We now study the trading of patents between producing firms and the NPE.

Proposition 1. *Under Assumptions 1-5, the NPE will buy exactly one patent if one firm discovered both components, and none otherwise. In that case, the NPE's effect on equilibrium payoffs is to decrease the payoff of the firm that has no patents.*

Proof. The details of the proof are in Appendix B. When we consider Games 1, A and D, under our assumptions the NPE makes TIOLI offers and buys one patent from the firm that owns both components. The price is such that firm 1 is indifferent between selling a patent and keeping both.[23] The NPE's payoff is strictly positive and equal to $\pi_{NPE} = \beta^2 F + c - \beta(1-\beta)I\pi_d$, while the payoff of firm 1 remains the same. The payoff of firm 2 is reduced relative to its payoff in the absence of an NPE. We find that this is the only case in which the NPE affects the equilibrium payoffs.

In Games 2, B, and C, it is clear that the NPE can at best (and at worst) break even by buying patents and licensing. In particular, no firm can increase its own and the NPE's joint profit by trading patents. So regardless of how much trade occurs, the final payoffs in all these three cases are $\pi_1 = \pi_d$, $\pi_2 = \pi_d$, $\pi_N = 0$. $\qquad \square$

Intuition for this result: Although the NPE is endowed with the same capacity to litigate as a producing firm, we are able to find a mechanism through which the NPE makes positive profits, in the case where one firm discovers both components. Imagine that firm 1 owns the patents for both components. Because of the cost savings in trials for multiple components, plus the fact that suing in separate trials is not a credible threat, the license reflects firm 2's willingness to pay up to avoid the litigation cost c. However, when firm 1 sells one patent to the NPE, firm 2 now faces a credible litigation threat from two different parties. Firm 2 will now buy licenses from both parties to avoid the litigation cost c from each one of

---

[22]We analyzed each game for an arbitrary set of parameters. These computations have been omitted for the sake of exposition, but are available upon request.

[23]Notice that the NPE can at best break even if it buys both patents at a price that keeps firm 1 indifferent. That is, the NPE and firm 1 cannot increase their joint profits if they trade both patents.

them. Hence, unbundling patent ownership increases the total surplus that can be extracted from firm 2. This is the *patent privateering effect*. A second way that the NPE makes positive profits is by charging the fee F which is paid once per product to the suing firm. If firm 2 infringes on both patents, which occurs with probability β^2, the damages award per product entitles firm 1 to a payment of F. However, when the patents are owned by both firm 1 and the NPE and firm 2 infringes on both patents, the damages award per product F has to be paid to two parties, increasing by F the willingness to pay for licenses. This is the *royalty stacking effect*.

These two effects combined allow the NPE to extract $c+\beta^2 F$ from licenses, which firm 1 would not extract if it kept both patents. The drawback of selling one patent is that the NPE cannot get an injunction. This decreases the willingness to pay of firm 2 for a license from the NPE. This effect is evident when only the component sold to the NPE is infringed by firm 2 (which occurs with probability $\beta(1-\beta)$). Therefore, the condition $\beta^2 F + c > \beta(1-\beta)I\pi_d$ guarantees that the extra licensing revenue that can be obtained by disaggregating the patent portfolio is larger than the loss in the license fee by the NPE due to the lack of injunctions.

6.2   The Research Stage

We can now see what is the effect of the NPE on the research efforts. The impact of the NPE is to increase the difference between winning and losing for the firm that is behind. As a consequence, the firm without discoveries after the first stage exerts more effort to discover the second component when the NPE exists. In equilibrium, the extra effort of firm 2 impacts the effort of firm 1 in such a way that aggregate effort always increases. Going back to the initial stage, the continuation values again increase the difference between being the first firm to discover some component and not. Thus, the equilibrium effort in the first stage also increases. We summarize the main result of the section in the following proposition:

Proposition 2. *The effect of an NPE is to increase the aggregate effort in the research stage.*

1. *When nothing has been discovered, both firms increase their effort uniformly on each component.*
2. *When one component is discovered, the firm that has no discoveries puts more effort to discover the second component. The firm that made the discovery can increase or decrease its effort (compared to the case of no NPEs). However, aggregate effort always increases.*

The proof comes from a series of Lemmas which we prove in the following two subsections.

6.2.1   Intermediate Stage: One component discovered

In this subsection we assume without loss of generality that firm 1 made the first discovery. Under our assumptions about litigation, entry and licensing, the continuation values for the game *without the NPE* are as follows.

- If firm 1 discovers the second component, we have the following:

  Firm 1's continuation value: $\equiv V_1(2) = \pi_d + \beta(2 - \beta)(F + I\pi_d) + 2\beta R + c.$

  Firm 2's continuation value: $\equiv V_2(0) = \pi_d - \beta(2 - \beta)(F + I\pi_d) - 2\beta R - c.$

- If firm 2 discovers the second component, both firms get duopoly profits: $V = \pi_d.$

In the game *with an NPE*, the only difference arises when firm 1 discovers both components. In this case the NPE obtains positive rents from firm 2. Thus the only change under an NPE is the reduction in the continuation value for firm 2 by $\bar{\theta} = \beta(F + R) + c.$

Thus, the effect of the NPE can be analyzed as the effect of the change of one parameter in a game. The parameter would effectively take only two values: $\theta = 0$ representing the absence of NPEs and $\theta = \bar{\theta} > 0$ representing the effect of an NPE. However, we develop a comparative static result for any $\theta \in [0, \Theta]$.

The decision for the firms is how much effort to exert to discover the remaining component. We focus on a pure strategy equilibrium, that is, firm i chooses effort $u_i \in [0, U]$, taking the effort choices of firm 2 as given. We model innovation as a Tullock contest, which is equivalent to a patent race without discounting (See for example Corchón (2007)). We assume that effort is costly and both firms have the same cost function $c(u) = \frac{u^2}{2}$. This assumption is not crucial and can be generalized to any increasing convex cost function, using the comparative static result from Acemoglu and Jensen (2013).

The firms solve:

$$V^*(\theta) \equiv \max_{u_1} \frac{u_1 V_1(2) + u_2 V}{u_1 + u_2} - c(u_1).$$
$$V^{**}(\theta) \equiv \max_{u_2} \frac{u_2 V + u_1(V_2(0) - \theta)}{u_1 + u_2} - c(u_2).$$

Since the objective functions are strictly concave and differentiable, the best response functions are well defined and they are characterized by the first order conditions:

$$\frac{u_2(V_1(2) - V)}{(u_1^*(u_2) + u_2)^2} = u_1^*(u_2). \quad \text{(FOC 1)}$$

$$\frac{u_1(V - V_2(0) + \theta)}{(u_1 + u_2^*(u_1))^2} = u_2^*(u_1). \quad \text{(FOC 2)}$$

Define $L = \beta(2 - \beta)(F + I\pi_d) + \beta R + c$, which is the license that the firm with two discoveries can charge to the firm that enters the market without any patents.

Lemma 1. *When $\theta = 0$ (NPE does not exist), there are two equilibria: $(u_1, u_2) = (0, 0)$ and $(u_1, u_2) = \frac{1}{2}(\sqrt{L}, \sqrt{L})$.*

Notice that in these equilibria there is no *discouragement effect*; the firm who is the current leader will put the same effort towards the discovery of the second component as the firm that is behind. The result

comes from the fact that one of the firms will make a discovery for sure. If we add a small probability that none of the firms make the discovery, the firm that is ahead will put more effort than the firm that is behind.

Lemma 2. *When $\theta > 0$ (NPE exist), there are two equilibria: $(u_1, u_2) = (0,0)$ and $(u_1, u_2) > 0$, where $u_2 = Ku_1$ and $K > 1$.*

Remark: The equilibrium aggregate effort, when equilibrium effort is positive, is increasing in θ. In an equilibrium where there is positive effort exerted by the firms, the aggregate effort is given by

$$u_1 + u_2 = \sqrt[4]{(L + \theta)L}.$$

This result extends with much more generality than our specification. However, we chose the simple specification for tractability in the next section.

6.2.2 First Stage: Nothing yet discovered

When nothing has been yet discovered, firms are symmetric and each firm decides how much effort to allocate to discovering components x and y. Denote firm i's efforts by u_i^x, u_i^y. Similarly to the previous section, the optimal effort choices are given by the solution to

$$\max_{u_i^x, u_i^y} \frac{V^*(\theta)(u_i^x + u_i^y) + V^{**}(\theta)(u_{-i}^x + u_{-i}^y)}{r + u_i^x + u_i^y + u_{-i}^x + u_{-i}^y} - c(u_i^x) - c(u_i^y)$$

Lemma 3. *The difference $D(\theta) = V^*(\theta) - V^{**}(\theta)$ is positive for all θ and $D(\theta) > D(0)$, for all $\theta > 0$.*

Lemma 4. *If $V^* \geq V^{**}$ there is a unique symmetric equilibrium where $u_x = u_y = U^*$.*

The first result in Lemma 3 implies (by Lemma 4) the existence of a unique symmetric equilibrium. The second conclusion in the Lemma 3 is that the difference in payoffs between winning and losing is larger with an NPE than without an NPE. Therefore, the effect of the NPE in the continuation game rolls back all the way to the initial research stage, implying that firms will exert more effort than they would have exerted in the absence of the NPE.

6.3 Entry Deterrence

One reason why NPEs might have a negative effect on the rate of innovation is entry deterrence. As our model illustrates, the reason why NPEs exist is because they can, jointly with entities which engage in R&D, extract surplus from firms which want to enter the product market but lack the patents necessary to protect themselves against litigation. If industry profits are high enough to sustain such firms, they

will enter the product market even at the lower levels of profit driven by the NPE.[24] Here we study what happens if this is not the case—and instead NPEs effectively deter the follower firm.

First, it is important to establish a benchmark against which to compare NPEs. Notice that if the entry condition fails, that is,

$$\pi_d < \frac{\beta(2-\beta)F + 2\beta R + c}{1 - \beta(2-\beta)I},$$

then the firm that made both discoveries can deter the rival firm from entering, even in the absence of an NPE. Thus, in this case entry deterrence occurs by the very nature of the patenting and litigation system, in a world where NPEs did not exist.

So instead we ought to focus on the case where a competitor would enter the product market in the absence of NPEs, but would be deterred from entering by an NPE. The relevant subgames of our model are those where firm 1 (wlog) patents both components: games 1, A and D. First, it is easy to see that subgame A is in fact irrelevant for the same reasons as before: because firm 1 would never want to sell its entire patent portfolio to the NPE, since this would not produce a larger threat of litigation, and would in fact remove the possibility of an injunction against firm 2. We can now focus on subgames 1 and D.

As discussed above, the NPE acts as an entry deterrent if in its absence firm 2 would enter the final product market, whereas in its presence firm 2 would not enter. This is the case when

$$\frac{\beta(2-\beta)F + 2\beta R + c}{1 - \beta(2-\beta)I} \le \pi_d \le \frac{2\beta(F + R) + 2c}{1 - \beta I}$$

Notice that the region between these two bounds is non-empty if and only if

$$c \ge \frac{2\beta^2(1-\beta)I(F+R) - \beta^2(1-\beta I)F}{1 - 3\beta I + 2\beta^2 I}$$

For the remainder of this section we assume that this condition holds, in addition to Assumptions 1, 2, 3, and 5.[25] These simply say that industry profits are higher under a monopoly regime, that litigation is a credible threat, and that, all else equal, entities prefer to licence rather than litigate if they can extract the same surplus.

When firm 1 is deciding whether to sell one of its patents to the NPE or not, it chooses between two continuation games: game 1, where equilibrium payoffs are

$$\pi_1 = \pi_d + L \qquad \pi_2 = \pi_d - L \qquad \pi_{NPE} = 0$$

with $L = \beta(2-\beta)(F + I\pi_d) + 2\beta R + c$, and game D, where the equilibrium payoffs are

$$\pi_1 = \pi_m + p_1 \qquad \pi_2 = 0 \qquad \pi_{NPE} = 0 - p_1$$

The latter is the case where by selling one of its patents to the NPE, firm 1 drives firm 2's potential profits so low that firm 2 does not enter. To have trade in patents we require firm 1 to be willing to sell and the

---

[24]This is the case when our Assumption 4 holds, corresponding to the entry condition in Game D.

[25]These condition are compatible as for $I = 0$ they can all be satisfied simultaneously.

NPE willing to buy. Therefore, we require

$$\pi_d + L - \pi_m \le p_1 \le 0 \qquad \Leftrightarrow \qquad \pi_d + \beta(2 - \beta)(F + I\pi_d) + 2\beta R + c - \pi_m \le p_1 \le 0.$$

Thus, when π_m is very large, it is possible to have trade with $p_1 \le 0$. Otherwise, the NPE simply would not buy the patent. Firm 1 needs to be willing to pay to transfer the patent to the NPE. This might seem strange, but there is evidence that it occurs. Patent privateering is exactly this condition: firms pay or transfer part of their portfolio to other entities, notably non producing entities, to enforce their patents.

The analysis above shows that NPEs may allow patent holders to leverage the patent system and obtain larger profits than in the absence of NPEs. Evaluating the welfare effect of NPEs is a more subtle issue than merely calculating their effects on equilibrium profits. First, consider the original purpose of the patent system—to prevent competitors from appropriating the rents that are necessary for a firm to produce innovation in the first place. In our model a single firm with patents on both components x and y may be able to deter entry on its own, provided the profit of a potential entrant is negative. But as we have shown, NPEs lower the profits of such potential entrants and hence enhance the ability of the inventor to deter entry, above and beyond the extent to which the patent system itself may have been designed to allow the incumbent to do this. In other words, if one thinks that the patent system is optimally designed to reward inventors with some ability to exclude competitors, the effect of NPEs is to strengthen this ability to a level above the optimal one, and thus to provide super-optimal rewards to patenting.

This phenomenon may decrease overall welfare through multiple different channels. First, consumer surplus will likely be lower under a monopoly than under a duopoly. In the case where an NPE allows a firm to sustain a monopoly which it would not be able to sustain otherwise, the overall effect of an NPE on welfare may be negative, even though the incentives for innovation may be higher. Second, it may be the case that social surplus is larger when multiple firms, rather than a single one, produce innovations, because of spillovers or because of learning-by-doing.

7 Extensions

7.1 Selling to a third producing firm

In our main model we showed that trade in patents between firms and the NPE occurs only when one of the firms owns patents for both component. The optimal strategy is to sell one of them to the NPE in order to extract a larger surplus from licenses from the firm that does not own patents. We now study the case where the third party is a producing firm which does not do research, rather than an NPE.

There is a strategic difference between selling patents to a firm with the ability to enter the product market rather than an NPE. The latter operates under the commitment of not entering the product market, while the former could enter the market and become a competitor. Another difference is the prices that a

producing firm is willing to pay to acquire the patent, because the size of the license it can extract from litigation differs from what the NPE can get. This difference is mostly due to injunctions and due to the fact that market structure changes when the third party enters the product market.

We explore how these differences operate and we find conditions under which a producing firm is willing to pay more than the NPE to acquire a patent. To make the analysis clear, we focus on the set of conditions that guarantee that all firms enter the market, litigation is a credible threat, and licenses are accepted in equilibrium. We explore how these differences operate and we find conditions under which a producing firm is willing to pay more than the NPE to acquire a patent. To make the analysis clear, we focus on the set of conditions that guarantee that all firms enter the market, litigation is a credible threat, and licenses are accepted in equilibrium.

The third party is labeled as 'firm 3', and replaces the role of the NPE in that it does not put effort to research, but can acquire patents from firms 1 and 2, sell licenses after the acquisition, and it can enter the product market. Suppose firm 3 acquired the patent of component x from firm 1. After the acquisition, firms 1, 2 and 3 entered the market. Since firm 2 does not own any patents, and firm 3 only owns the patent for one component, there is a licensing stage. Firm 1 offers simultaneously the licenses L_2^y and L_3^y to firm 2 and firm 3, respectively. Simultaneously, firm 3 offers a license L^x to firm 2. Given these offers, firm 2 and firm 3 accept or reject them. Since firm 2 and firm 3 do not observe all the decisions, we assume passive beliefs: upon a deviation from equilibrium offers, firms believe that the offers for their rivals haven't changed.

Proposition 3. *When firm 1 owns patents for both components, there is trade in patents between firms 1 and 3 if*

$$c + \beta^2 F \geq (2 + (1 - \beta)^2)(\pi_d - \pi_T) + \beta(1 - I)[2 - \beta(1 + I)]\pi_T.$$

Moreove, firm 1 would rather sell the patent for one component to a practicing entity rather than to an NPE iff

$$(2 - \beta I)\pi_T \geq \pi_d$$

Proof. We index the licenses offers as (L^x, L_2^y, L_3^y) and the accept/reject decisions accordingly. Let π_T denote triopoly profits, $K^+ = \pi_T - c + \beta(F + R)$ and $K^- = \pi_T - c - \beta(F + R)$. Appendix D contains the details of the payoffs after each continuation of acceptance and rejection of the license contracts.

We want to find conditions to sustain (A,A,A) as an equilibrium with passive beliefs. Firm 3 compares the payoff from (A,A,A) with (A,A,R). The maximal license fee that firm 1 can extract is $L_3^y = \beta(F + R + I\pi_T) + c := \alpha_T$. Firm 2, must prefer (A,A,A) over (R,A,A), (A,R,A) and (R,R,A). With passive beliefs, firm 2 assumes firm 1's offer to firm 3 does not change, and it is therefore accepted. Hence, we require: $\max\{L_2^y, L^x\} \leq \beta[F + R + I\pi_T] + c$ and $L_2^y + L^x \leq 2\beta(F + R) + 2c + \beta I(2 - \beta I)\pi_T$. There are many ways to divide the surplus. Since we have multiple ways of achieving the equilibrium, let's assume that firm 1 charges $L_2^y = f \in [\alpha_T - (\beta I)^2 \pi_T, \alpha_T]$ and firm 3 charges $L^x = S_p - f$. These offers will be credible if firms

32

1 and 3 are willing to make them, rather than litigate. Therefore, we require firm 3's payoff from (A,A,A) to be larger that its payoff under (R,A,A). A sufficient condition for this to happen (when f is as large as possible, $f = \alpha_T$, so $L^x = S_p - \alpha_T$) is

$$2c \geq \beta I(\pi_d - (2 - \beta I)\pi_T)$$

Also, we require that firm 1's offers are credible. Therefore, we need (A,A,A) preferred to (A,R,R), (A,R,A) and (A,A,R), from firm 1's perspective. Sufficient conditions for this to happen (Assuming that f is as small as possible, $f = S_P - \alpha_T$, so the payoff of (A,A,A) is exactly $S_p + \pi_T$) are:

The condition for (A,A,A) preferred to (A,R,R) is $4c \geq \beta I[\beta I(\pi_M - \pi_d) + 2\pi_d - \pi_T]$.
The condition for (A,A,A) preferred to (A,R,A) is $2c \geq \beta I(\pi_d - (2 - \beta I)\pi_T)$.
The condition for (A,A,A) preferred to (A,A,R) is $2c \geq \beta I(\pi_d - 2\pi_T)$.

It can be shown that the condition $4c \geq \beta I[\beta I(\pi_M - \pi_d) + 2\pi_d - \pi_T]$ implies the rest of the inequalities. Suppose this condition holds, so (A,A,A) is an equilibrium for any equilibrium in which firm 1 obtains f from the surplus. Then, the equilibrium payoffs are:

$$\pi_1 = \pi_T + \alpha_T + f, \qquad \pi_2 = \pi_T - S_p, \qquad \pi_3 = \pi_T + S_p - \alpha_T - f$$

Consider the worst equilibrium outcome for firm 1, which corresponds to $f = S_p - \alpha_T$. In that case, the equilibrium payoffs are:

$$\pi_1 = \pi_T + S_p, \qquad \pi_2 = \pi_T - S_p, \qquad \pi_3 = \pi_T.$$

Will firm 1 sell to firm 3 even in this case? By not selling firm 1 gets a payoff of $\pi_d + \hat{\alpha}$. Therefore, firm 1 sells one patent to firm 3 iff:

$$c \geq (2 + (1 - \beta)^2)(\pi_d - \pi_T) + \beta(1 - I)[2 - \beta(1 + I)]\pi_T - \beta^2 F.$$

Notice the difference with the NPE case, where the maximal joint license fee that firm 1 and the NPE extracted from firm 2 was $2\beta(F + R) + 2c + \beta I\pi_d$. Two producing entities can extract $S_p = 2\beta(F + R) + 2c + \beta I(2 - \beta I)\pi_T = 2\alpha_T - (\beta I)^2\pi_T$. Therefore, firm 1 and 3 can extract more from firm 2 than firm 1 and the NPE iff $(2 - \beta I)\pi_T \geq \pi_d$. □

7.2 Nash Bargaining over Licences

In this section we extend our model of patent licencing to include bargaining power. Specifically, licencing happens in the shadow of litigation, as in our main model, but the hypothetical plaintiff and defendant have bargaining powers b and $1 - b$, respectively, over the surplus that a licencing agreement generates, relative to the players' litigation payoffs. Our analysis so far has consistently assumed that the plaintiff in every potential lawsuit is always able to make a take-it-or-leave-it licence offer, which is equivalent to the special case of $b = 1$. Hence this section generalizes our main result to accommodate the possibility that a

patent-holder who threatens to litigate may not capture the entire surplus resulting from licencing instead of litigating. Despite the vast discussion about litigation (Spier (2007)), there is no clear evidence that the plantiff has more or less bargain power than the defendant. Therefore, our analysis is one of comparative statics over the bargain parameter. Throughout the analysis we will focus on the case where litigation is a credible threat, players prefer to licence rather than litigate if they can extract at least as much surplus, and entry is profitable for the producing firms.

Game 1

We focus on the case where an agreement means that both licenses are accepted and a disagreement means that no license is accepted. Under the conditions assumed before, when an agreement is not reached firm 1 will sue firm 2 for infringing both components. Using the Nash bargain solution, we obtain that the optimal licenses for the components are such that:

$$L_1^* + L_2^* = \hat{\alpha} - (1 - b)[2c - \beta(2 - \beta)I(\pi_m - \pi_d)].$$

Notice that condition (Lic1) implies $2c - \beta(2 - \beta)I(\pi_m - \pi_d) \geq 0$. Under this condition, it is also true that for any value of the bargain parameter b firm 1 will always prefer to license over suing. The entry condition for firm 2 is now easier to satisfy, because it can extract some surplus in the negotiation. Therefore, in this case, the equilibrium payoffs are: $\pi_1 = \pi_d + \hat{\alpha} - (1 - b)[2c - \beta(2 - \beta)I(\pi_m - \pi_d)], \quad \pi_2 = \pi_d - \hat{\alpha} + (1 - b)[2c - \beta(2 - \beta)I(\pi_m - \pi_d)], \quad \pi_{NPE} = 0.$

Game 2

Consider the continuation game where firm 1 holds a patent on x and firm 2 holds a patent on y, and consider the history where both firms have entered the final product market. As in the benchmark model, we assume that if the firms do not cross-licence, it is a dominant strategy for each of them to sue. If the firms were to instead offer each other licences, the holder of each patent would be able to extract a b share of the additional surplus generated by avoiding litigation on that patent. Since the firms' patents on x and y are symmetric, and each holds one, they each capture half of the total surplus from cross-licencing both patents instead of litigating on both (b share from the firm's own patent and $1 - b$ from the rival's patent). So surplus is divided evenly and the final payoffs are: $\pi_1 = \pi_d, \quad \pi_2 = \pi_d, \quad \pi_{NPE} = 0.$

Game A

In this game the NPE bought both patents from firm 1. As in Game 1, we focus on the case where an agreement means that both licenses are accepted and a disagreement means that no license is accepted. Under the conditions assumed before, when an agreement is not reached firm 1 will sue firm 2 for infringing

both components. Using the Nash bargain solution, we obtain that the optimal licenses for the components are such that:

$$L_x^* + L_y^* = \beta[(2 - \beta)F + 2R] + (2b - 1)c.$$

Just like in the baseline model, where the NPE had all the bargain power, the NPE always prefers to sell licenses rather than litigate. Firm 2, on the other hand, obtains larger payoffs now compared to the baseline model. Thus the entry condition will be easier to satisfy. Hence in equilibrium: $\pi_1 = \pi_d + p_1 + p_2$, $\quad \pi_2 = \pi_d - \beta[(2 - \beta)F + 2R] - (2b - 1)c$, $\quad \pi_{NPE} = \beta[(2 - \beta)F + 2R] + (2b - 1)c - p_1 - p_2$.

Game B

Consider the continuation game B where firm 1 discovered x, firm 2 discovered y, and the NPE holds both patents, and consider the history where both firms 1 and 2 have entered the final product market. Under Assumption 2 the NPE can credibly threaten to sue each firm on the component that it did not discover, if the firm rejects a licence offer. The NPE thus has two hypothetical lawsuits that it could bring. In each lawsuit, the defendant's expected payoff would be $\pi_d - \beta(F + R) - c$, while the plaintiff's incremental payoff from that lawsuit would be $\beta(F + R) - c$. If the NPE and the firm were to instead agree on a licence L, the Nash bargaining solution would require that the plaintiff captures b share of the surplus generated from avoiding litigation, which is $2c$ in total, while the defendant captures the remaining $1 - b$. Hence $L = \beta(F + R) - c + b(2c)$. So the final payoffs are: $\pi_1 = \pi_d - \beta(F + R) - (2b - 1)c + p_1$, $\quad \pi_2 = \pi_d - \beta(F + R) - (2b - 1)c + p_2$, $\quad \pi_{NPE} = 2[\beta(F + R) + (2b - 1)c] - p_1 - p_2$.

Notice that the new entry condition for both firms to enter the product market will be

$$\pi_d \geq \beta(F + R) + (2b - 1)c$$

Game C

Consider the continuation game C where firm 1 discovered x, firm 2 discovered y, the NPE holds the patent on x, and firm 2 holds the patent on y, and consider the history where both firm 1 and firm 2 have entered the product market. Under Assumption 2 both firm 2 and the NPE can credibly threaten to litigate. Instead, the NPE may offer a licence L_N to firm 2, and firm 2 may offer a licence L_2 to firm 1. We can now find what licence price corresponds to bargaining power b for the hypothetical plaintiff and $1 - b$ for the hypothetical defendant in each lawsuit.

Consider the licence L_2 which firm 2 will offer to firm 1, based on the patent for component y. If firm 2 were to sue, the payoffs would be

$$\pi_1 = (1 - \beta I)\pi_d - c - \beta(F + R), \qquad \pi_2 = (1 - \beta I)\pi_d + \beta I \pi_m - c + \beta(F + R) - L_N$$

If they agree on a licence L_2, the payoffs will be $\pi_1 = \pi_d - L_2$, $\qquad \pi_2 = \pi_d - L_N + L_2$. So the incremental surplus generated by avoiding litigation is $2c - \beta I(\pi_m - 2\pi_d)$. By assumption 3 this term is positive and

we have $L_2 = \beta(F + R + I\pi_d) + c - (1 - b)[2c - \beta I(\pi_m - 2\pi_d)]$. Next, consider the licence L_N that the NPE will offer to firm 2, based on the patent that it holds for component x. If the NPE were to sue, the payoffs would be

$$\pi_2 = \pi_d + L_2 - \beta(F + R) - c, \qquad \pi_{NPE} = \beta(F + R) - c$$

If they agree on a licence L_N, the payoffs will be $\pi_2 = \pi_d + L_2 - L_N, \qquad \pi_{NPE} = L_N$. So the incremental surplus generated by avoiding litigation is $2c$. We thus have $L_N = \beta(F + R) + c - 2c(1 - b)$. Hence:

$\pi_1 = \pi_d - \beta(F + R + I\pi_d) + c - (1 - b)[2c - \beta I(\pi_m - 2\pi_d)] + p_1$,

$\pi_2 = \pi_d + L_2 - L_N = \pi_d + \beta I\pi_d + (1 - b)\beta I(\pi_m - \pi_d), \ \pi_{NPE} = \beta(F + R) + c - 2c(1 - b) - p_1$.

Game D

In this game, firm 1 made both discoveries but sold one of its patents to the NPE. Because there is no interaction among lawsuits we can treat them as two independent problems. Then the equilibrium payoffs are:

$$\pi_1 = \pi_d(1 + \beta I) + \beta(F + R) + (1 - b)\beta I(\pi_m - 2\pi_d) + (2b - 1)c + p_1$$

$$\pi_2 = \pi_d(1 - \beta I) - 2\beta(F + R) - (1 - b)\beta I(\pi_m - 2\pi_d) - 2(2b - 1)c$$

$$\pi_{NPE} = \beta(F + R) + c + 2(b - 1)c - p_1$$

Even when the NPE has zero bargain power ($b = 0$), selling a license is profitable by Assumption 2.

To summarize the previous results consider the following notation:

$$\lambda(b) = (1 - b)[2c - \beta(2 - \beta)I(\pi_m - \pi_d)], \quad H(b) = (1 - b)[2c - \beta I(\pi_m - 2\pi_d)], \quad G(b) = 2c(1 - b).$$

These quantities denote the amount of surplus that a firm buying the license can collect in the bargaining process. They are all positive under our assumptions on the parameters. When $b = 1$, the buyer has no bargain power. As b moves towards zero the buyer of the license is able to extract positive surplus from the negotiation.

The way in which the bargaining power affects the payoffs depends on who is making the offer, due to the injunctions. Denote by π_i^G the equilibrium payoff of firm $i = 1, 2, N$ in game G in the benchmark model (TIOLI offers). Under our conditions, all firms buy licenses and there is no litigation on the equilibrium path. The payoffs are given by:

Game 1:	Game A:	Game D:
$\pi_1 = \pi_1^1 - \lambda(b)$	$\pi_1 = \pi_1^A$	$\pi_1 = \pi_1^D - H(b)$
$\pi_2 = \pi_2^1 + \lambda(b)$	$\pi_2 = \pi_2^A + G(b)$	$\pi_2 = \pi_2^D + H(b) + G(b)$
$\pi_{NPE} = \pi_{NPE}^1$	$\pi_{NPE} = \pi_{NPE}^A - G(b)$	$\pi_{NPE} = \pi_{NPE}^D - G(b)$

Game 2:   **Game B:**   **Game C:**

$$\pi_1 = \pi_1^2 = \pi_d \qquad\qquad \pi_1 = \pi_1^B + G(b) \qquad\qquad \pi_1 = \pi_1^C + H(b)$$

$$\pi_2 = \pi_2^2 = \pi_d \qquad\qquad \pi_2 = \pi_2^B + G(b) \qquad\qquad \pi_2 = \pi_2^C - H(b) + \lambda(b)$$

$$\pi_{NPE} = \pi_{NPE}^2 = 0 \qquad\qquad \pi_{NPE} = \pi_{NPE}^B - 2G(b) \qquad\qquad \pi_{NPE} = \pi_{NPE}^C - \lambda(b)$$

Notice that in Game 1, firm 1 losses surplus in the negotiation in both components. Selling one patent to the NPE might be convenient if the NPE loses less in the negotiation than firm 1. However, the NPE can extract only some surplus through the license fee, since the NPE does not get an injunction. In the benchmark case the NPE never bought the two patents because it would lose money. Here, it is even worse, since it loses more money in the negotiation. However, in the benchmark case, it was optimal to sell one component to the NPE to extract more surplus from firm 2. With bargain power, firm 1 loses money keeping both patents, but also the NPE loses money negotiating it. Therefore, there is trade of one patent if and only if:

$$\beta R + \beta(1-\beta)[F + I\pi_d] + H(b) - \lambda(b) \le p_1 \le \beta(F + R) + c - G(b).$$

This region is not empty as long as

$$c + \beta^2 F - \beta(1-\beta)I\pi_d + \lambda(b) - G(b) - H(b) \ge 0$$

which is equivalent to

$$c + \beta^2 F - \beta(1-\beta)I\pi_d \ge (1-b)[\beta(1-\beta)I(\pi_m - \pi_d) + \beta I\pi_d + 2c]$$

By assumption 5, this condition holds for $b = 1$. But, we can see that for $b < 1$ this condition could be violated. In that case, there is no trade in patents. The NPEs will not play a role in the case of a single owner.

8 Conclusion

This paper primarily considers the question: how do non-practicing entities change the incentives for innovation? To answer this question we study a model where: 1) firms first invest effort in research, which determines their subsequent patent portfolios; 2) firms (including NPEs) then trade in patents, engage in licensing, and litigate against alleged infringers. Our model captures many important features of today's intellectual-property-based economy, particularly ones which are most salient in the industries where NPEs have emerged as very active participants. First, we consider patents for innovations which do not cover the entirety of a product. Instead, a product in our model consists of multiple components which may be discovered and patented by different entities. Second, patents themselves are inherently uncertain–if a firm initiates litigation the outcome of the suit is random and the probability of success is a function of the firm's total patent portfolio. This captures the idea that patents in many industries tend to have fuzzy

boundaries and therefore litigation outcomes may be less predictable. Third, we embed our discussion of trade, licensing and litigation into an ex-ante innovation race, where firms decide how much to invest in new discoveries, anticipating the future rewards that accrue from product sales and patent trade and licensing. This R&D stage is essentially a patent race for multiple patents, similar to the standard patent race model.

We focus on one particular kind of NPE, among many that exist in reality, so we cannot claim that our results generally apply to all. For example, we abstract away from the issues of weak patents and nuisance litigation. Instead we model strong, but probabilistic patents, which can credibly be litigated. Our model is also one where, by design, the NPE has no outside source of patents. The only source of innovation in our economy are firms which carry out R&D and then have the option to produce a product. In this sense, all innovation in our model is endogenous and is determined directly by the rewards stemming from obtaining patents.

Our model allows us to study the effect of NPEs by comparing two cases: a benchmark economy where NPEs do not exist (or cannot litigate as a producing entity can), and an economy where NPEs exist, and may buy patents, license them, and sue infringers, just as a producing firm would. We find that NPEs change the the incentives of innovators in a very particular way. First, when firms emerge from the research stage with symmetric patent portfolios, NPEs can at best earn zero profits and do not affect the continuation payoffs of the producing firms in the subsequent patent trade and litigation subgames. On the other hand, when the outcome of the R&D process is that one firm makes more discoveries than the other, and therefore has a larger portfolio, we find that the NPE indeed affects the equilibrium payoffs of the producing firms. We fully characterize this effect by looking at all the different possible cases, depending on the parameters of the model: the magnitude of legal costs (c), the structure of industry profits (π_m and π_d), the uncertainty of litigation outcomes (β and I), and the rewards of successful plaintiffs (F and R).

In the latter case (with asymmetric patent portfolios), we find that generally the effect of the NPE is to widen the gap between the equilibrium profit of the 'leader' firm and that of the 'losing' firm. Therefore ex ante rewards to innovation increase, since the marginal value of obtaining a patent increases. In this case the NPE may make positive profits, and this is entirely at the expense of the "follower" firm. That is, the NPE does not affect the continuation payoff of the "leader" firm, which emerges from the research stage with a larger patent portfolio. But the NPE does lower the continuation payoff of the follower, because it can credibly threaten to inflict upon it larger legal costs, which in the absence of an NPE the leader firm could not do. Since these higher costs are credible, the NPE can extract more surplus from the follower firm in a licensing deal. Thus the firm which ex post turns out to be the follower is worse off when an NPE is active in the economy.[26] As our model shows, what matters is not just the levels of equilibrium profits, but also the margin between firms' profits, which drives the incentives for patenting.

[26]Moreover, the legal mechanism through which the NPE changes the equilibrium outcome does not depend on any particular assumptions that favor an NPE, as would be the case if we had assumed that an NPE has lower litigation or search costs than a producing firm.

Our analysis also shows that while the NPE generally increases the incentives for innovation, in some circumstances it may do so in undesirable ways, by acting as an entry deterrent and effectively allowing the leader firm to monopolise the product market. Notice that this monopolisation may happen even in the absence of any NPEs, for some parameter values of the model. However, the NPE may allow one firm to monopolise the market for a strictly larger set of parameter values, and hence the overall effect of the NPE on welfare may be negative, because it expands the set of circumstances where one firm dominates the market.

Although our main model assumes that patent holders make take-it-or-leave-it licence offers to other firms, we also extend the results to the more general setting where each agent has some bargaining power in the licence and patent trade stages. Our results are robust in this sense, and remain qualitatively unchanged. We also study the different incentives of an NPE and of a hypothetical third firm which may engage in the same sort of patent trade and licencing. We show that a patent owner might strictly prefer to sell a patent to the NPE, rather than to another firm, if in the latter case it would enable the third firm to enter the product market, which it otherwise would not have done. In that sense selling to an NPE may be preferred because it increases the leader's profits without inducing additional entry in the product market.

Having analyzed the possible outcomes of the patent trade and litigation games, we go one step backward and study the incentives for innovation in the intermediate research stage, where there is already one discovery. Since the NPE does not affect a "leader" firm's continuation payoffs, but does lower a "follower" firm's payoffs, it increases the margin between winning and losing the last research stage. Therefore the incentive of the firm with no discoveries to invest effort in R&D increases in the presence of an NPE. As an equilibrium consequence, the total amount of effort in this stage increases, which implies a higher rate of innovation in the intermediate stage. Finally, going back to the initial research stage, we show that again the presence of an NPE increases the margin between having one patent or none. In the initial stage both firms are symmetric and therefore both increase their effort. The intuition for this result is quite transparent: what matters for the rate of innovation is not just the overall surplus that firms receive from their patents, but rather, the difference between being the firm which makes more discoveries and being the firm which makes fewer discoveries. We think this is an interesting, perhaps counter-intuitive, insight, which shows why a theoretical model of the role of NPEs is a valuable contribution to the literature.

References

Daron Acemoglu and Martin Kaae Jensen. Aggregate comparative statics. *Games and Economic Behavior*, 2013.

Elettra Agliardi and Unibo DSE. The fuzzy value of patent litigation under imprecise information. Technical report, 2009.

James Bessen and Michael James Meurer. *Patent failure: How judges, bureaucrats, and lawyers put*

innovators at risk. Princeton University Press, 2008.

James E Bessen and Michael J Meurer. Patent litigation with endogenous disputes. *The American economic review*, 96(2):77–81, 2006.

James E. Bessen and Michael J. Meurer. The direct costs from npe disputes. *Boston Univ. School of Law, Law and Economics Research Paper No. 12-34*, June 28. 2012.

Gary Biglaiser. Middlemen as experts. *The RAND journal of Economics*, pages 212–223, 1993.

Colleen V Chien. Of trolls, davids, goliaths, and kings: Narratives and evidence in the litigation of high-tech patents. *NCL Rev.*, 87:1571, 2008.

Luis Corchón. The theory of contests: a survey. *Review of Economic Design*, 11(2):69–100, September 2007.

Vincenzo Denicolò. Do patents over-compensate innovators? *Economic Policy*, 22(52):679–729, 2007. ISSN 1468-0327.

Tom Ewing. Indirect exploitation of intellectual property rights by corporations and investors: Ip privateering and modern letters of marque and reprisal. *Hastings Sci. & Tech. LJ*, 4:1, 2012.

Joshua S. Gans and Scott Stern. Is there a market for ideas? *Industrial and Corporate Change*, 19(3): 805–837, June 2010.

John M Golden. Patent privateers: Private enforcements historical survivors. *Harvard Journal of Law & Technology*, 26(2), 2013.

Andrei Hagiu and David B. Yoffie. The new patent intermediaries: Platforms, defensive aggregators, and super-aggregators. *Journal of Economic Perspectives*, 27, Number 1:45–66, Winter 2013.

Erik Hovenkamp. Predatory patent litigation. 2013.

Jeremy Howells. Intermediation and the role of intermediaries in innovation. *Research Policy*, 35(5):715 – 728, 2006.

Sara Jeruss, Robin Feldman, and Joshua H. Walker. The america invents act 500: Effects of patent monetization entities on us litigation. *Duke Law & Technology Review*, 11(3), 2012.

Mark Lemley and Carl Shapiro. Patent holdup and royalty stacking. 2006.

Mark A Lemley and Douglas Melamed. Missing the forest for the trolls, 2013.

Mark A. Lemley and Carl Shapiro. Probabilistic patents. *Journal of Economic Perspectives*, 19(2):75–98, Spring 2005.

Mark A. Lemley and Carl Shapiro. Patent holdup and royalty stacking. *Texas Law Review*, 85, 2007.

Josh Lerner. Trolls on state street?: The litigation of financial patents, 1976-2005. *Boston, MA: Harvard Business School, manuscript*, 2006.

Brian Love. An empirical study of patent litigation timing: Could a patent term reduction decimate trolls without harming innovators? *University of Pennsylvania Law Review, Forthcoming*, 2011.

Michael Mazzeo, Jonathan Hillel, and Samantha Zyontz. Excessive or unpredictable? an empirical analysis of patent infringement awards. *An Empirical Analysis of Patent Infringement Awards (June 17, 2011)*, 2011.

James F. McDonough. The myth of the patent troll: An alternative view of the function of patent dealers in an idea economy. *Emory Law Journal*, 56(07-6):189–230, 2006.

Shawn Miller. Do "fuzzy"software patent boundaries explain high claim construction reversal rates? 2012.

Tim Pohlmann and Marieke Opitz. Typology of the patent troll business. *R&D Management*, 43(2): 103–120, 2013.

Michael Risch. Patent troll myths. *Seton Hall Law Review - Villanova Law/Public Policy Research Paper*, 42, 2012.

John Rust and George Hall. Middlemen versus market makers: A theory of competitive exchange. (8883), April 2002.

David L. Schwartz and Jay P. Kesan. Analyzing the role of non-practicing entities in the patent system. *Cornell Law Review, Forthcoming*, 2012.

Fiona Scott Morton and Carl Shapiro. Strategic patent acquisitions. *Carl, Strategic Patent Acquisitions (July 2, 2013)*, 2013.

Carl Shapiro. Navigating the patent thicket: Cross licenses, patent pools, and standard setting. *Innovation Policy and the Economy*, pages 119–150, 2000.

Amol Sharma and Don Clark. Tech guru riles the industry by seeking huge patent fees. *Wall Street Journal*, September 27 2008.

Sannu K Shrestha. Trolls or market-makers? an empirical analysis of nonpracticing entities. *Colum. L. Rev.*, 110:114–2228, 2010.

Kathryn E Spier. Litigation. *Handbook of law and economics*, 1:259–342, 2007.

Daniel Spulber. Should business method inventions be patentable? *Journal of Legal Analysis*, 3(1): 265–340, 2011.

Appendix A: Analysis of equilibrium payoffs

We now analyze the effect of the NPE on equilibrium payoffs.

Single owner

First, consider the case where firm 1 (wlog) discovers both components x and y. This is the case where trade in patents can potentially occur, as long as the NPE and firm 1 can increase their joint profits by extracting surplus from firm 2. Suppose the NPE can make a TIOLI offer for one or both of firm 1's patents. There are three continuation games to consider: Game 1, Game A and Game D.

In what follows, we restrict attention to the case where litigation is profitable (i.e. it is a credible threat off the equilibrium path), entry in the final product market is profitable for the firms, and, all else being equal, all parties prefer to license as much as possible.

- *Game 1.* Under the assumptions:

$$\text{(Sue1)} \qquad c \leq \beta(F + R) + \beta I(\pi_m - \pi_d)$$

$$\text{(Lic1)} \qquad c \geq \frac{\beta(2 - \beta)I}{2}(\pi_m - 2\pi_d)$$

$$\text{(Entry1)} \qquad \pi_d \geq \frac{\beta(2 - \beta)F + 2\beta R + c}{1 - \beta(2 - \beta)I}$$

  The equilibrium of Game 1, where firm 1 owns both patents, has the following payoffs:

$$\pi_1 = \pi_d + \beta(2 - \beta)(F + I\pi_d) + 2\beta R + c$$

$$\pi_2 = \pi_d - \beta(2 - \beta)(F + I\pi_d) - 2\beta R - c$$

$$\pi_N = 0$$

- *Game A.* Under the assumptions:

$$\text{(Sue1')} \qquad c \leq \beta(F + R)$$

$$\text{(EntryA)} \qquad \pi_d \geq \beta(2 - \beta)F + 2\beta R + c$$

  The equilibrium of Game A, where the NPE buys both patents from firm 1, has the following payoffs:

$$\pi_1 = \pi_d + p_1 + p_2$$

$$\pi_2 = \pi_d - \beta(2 - \beta)F - 2\beta R - c$$

$$\pi_N = \beta(2 - \beta)F + 2\beta R + c - p_1 - p_2$$

- *Game D.* Under the assumptions:

$$\text{(Sue1)} \quad c \leq \beta(F + R) + \beta I(\pi_m - \pi_d)$$

$$\text{(Sue1')} \quad c \leq \beta(F + R)$$

$$\text{(Lic2)} \quad c \geq \frac{1}{2}\beta I(\pi_m - 2\pi_d)$$

$$\text{(EntryD)} \quad \pi_d \geq \frac{2\beta(F + R) + 2c}{1 - \beta I}$$

The equilibrium of Game D, where the NPE buys 1 patent from firm 1 and firm 1 keeps the other patent, has the following payoffs:

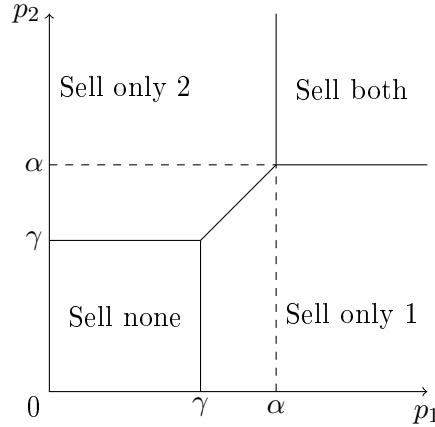$$\pi_1 = \pi_d + \beta(F + R + I\pi_d) + c + p_1$$

$$\pi_2 = \pi_d - \beta(2F + 2R + I\pi_d) - 2c$$

$$\pi_N = \beta(F + R) + c - p_1$$

Under the assumption that the NPE makes a take-it-or-leave-it offer, given the price p_k for the patent of component k, the decision of the firm that owns the patents is depicted below. The parameters are exactly those defined in Game 1,

$$\alpha = c + \beta[F + I\pi_d + R], \ \hat{\alpha} = c + \beta(2 - \beta)[F + I\pi_d] + 2\beta R, \ \gamma = \hat{\alpha} - \alpha = \beta R + \beta(1 - \beta)[F + I\pi_d].$$

Figure A.1. The decision to sell the patent for a component, given prices p_1 and p_2.

The NPE will decide its offers considering these regions. If it offers too little, the patent owner will not sell the patents. In that case, the NPE gets zero payoff. If the NPE sets $p_k \geq \gamma$ and $p_{-k} < \alpha$, the firm will only sell the patent for component k. In this case, the highest payoff the NPE can get is by buying the patent as cheap as possible: $\pi^*_{NPE,1} = \beta(F + R) + c - \gamma$. If the NPE sets $p_k > \gamma$ and $p_1 + p_2 \geq \hat{\alpha}$, the it will end up buying both patents. Therefore, the highest payoff the NPE can get is by buying the patent as cheap as possible: $\pi^*_{NPE,2} = \beta(2 - \beta)F + 2\beta R + c - 2\alpha$. Notice that the NPE loses money by buying both patents. Then, the only case where the NPE makes money is where it buys one of the patents, i.e.:

$$\pi^*_{NPE,1} > 0 \Leftrightarrow c + \beta^2 F > \beta(1 - \beta)I\pi_d.$$

The NPE can generate positive surplus iff the difference between what it can extract from firm 2 minus what firm 1 can extract is positive.

The left hand side $c + \beta^2 F$ represents the extra surplus the NPE can extract from firm 2. Because suing for both components bundles up the trial's cost, firm 2 saves in litigation costs an amount c. However, when firm 2 is sued by two parties, it has to pay the litigation cost twice. This is the first component of what the NPE can do and what firm 1 cannot commit to do in equilibrium. The second component relates to the infringement fee per product. Since these fees are awarded once per firm, the NPE can gather the extra fee F when both components infringe. If both components infringe (which occurs with probability β^2) firm 1 receives only F for the double infringement, while selling to the NPE allows them to joinlt extract $2F$ from firm 2. The right hand side is what firm 1 loses by selling to the NPE. If the patent sold to the NPE is the only one that infringes (which occurs with probability $\beta(1 - \beta)$), firm 1 will have forgone the opportunity to extract $I\pi_d$ from firm 2.

Multiple owners

Consider the case where the two components were patented by different firms, e.g. firm 1 discovers x and firm 2 discovers y. Again, the NPE will make TIOLI offers for the patents, and there are three continuation games to consider: Game 2, Game B and Game C.

As above, we restrict attention to the case where litigation is profitable (i.e. it is a credible threat off the equilibrium path), entry in the final product market is profitable for the firms, and, all else being equal, all parties prefer to license as much as possible.

- *Game 2.* Under the assumptions:

$$\text{(S-dom)} \qquad c \leq \beta(F + R + I(\pi_m - \pi_d)) - \beta^2 I(\pi_m - 2\pi_d)$$
$$\text{(CL-1)} \qquad c \geq \frac{\beta(1 - \beta)I}{2}(\pi_m - 2\pi_d)$$

  The equilibrium of Game 2, where each firm owns one patent, has the following payoffs: $\pi_1 = \pi_2 = \pi_d$ and $\pi_N = 0$

- *Game B.* Under the assumptions:

$$\text{(Sue1')} \qquad c \leq \beta(F + R)$$
$$\text{(EntryB)} \qquad \pi_d \geq \beta(F + R) + c$$

  The equilibrium of Game B, where the NPE buys both patents, has the following payoffs:

$$\pi_1 = \pi_d - \beta(F + R) - c + p_1$$
$$\pi_2 = \pi_d - \beta(F + R) - c + p_2$$
$$\pi_N = 2\beta(F + R) + 2c - p_1 - p_2$$

- *Game C.* Under the assumptions:

$$\text{(Sue1')} \qquad c \leq \beta(F + R)$$

$$\text{(Lic2)} \qquad c \geq \frac{\beta I}{2}(\pi_m - 2\pi_d)$$

$$\text{(EntryC)} \qquad \pi_d \geq \frac{\beta(F + R) + c}{1 - \beta I}$$

The equilibrium of Game C, where the NPE buys 1 patent from firm 1 and firm 2 keeps the other patent, has the following payoffs:

$$\pi_1 = (1 - I\beta)\pi_d - \beta(F + R) - c + p_1$$
$$\pi_2 = (1 + I\beta)\pi_d$$
$$\pi_N = \beta(F + R) + c - p_1$$

Notice that independently of firm j selling or keeping its patent, firm i will sell its own iff

$$p_i \geq \beta(F + R + I\pi_d) + c.$$

However, buying patents at these prices is not profitable for the NPE. At best, when $I = 0$, the NPE breaks even. Therefore, in equilibrium the NPE is never willing to buy patents and the final equilibrium payoffs are

$$\pi_1 = \pi_d \qquad \pi_2 = \pi_d \qquad \pi_N = 0,$$

and hence the NPE has no impact on the equilibrium payoffs when the two firms each make one discovery.

Remark: All of the assumptions that we have used above simplify to the following set of conditions (which correspond to assumptions 2-4 in the main text):

$$\text{(Sue1')} \qquad c \leq \beta(F + R)$$

$$\text{(Lic1)} \qquad c \geq \frac{\beta(2 - \beta)I}{2}(\pi_m - 2\pi_d)$$

$$\text{(EntryD)} \qquad \pi_d \geq \frac{2\beta(F + R) + 2c}{1 - \beta I}$$

To see this, notice that

(Sue1')$\Rightarrow$ (S-dom) $\Rightarrow$ (Sue1)

(Lic1)$\Rightarrow$ (Lic2)$\Rightarrow$ (CL-1),

(EntryD) $\Rightarrow$ (Entry1),(EntryA),(EntryB), (EntryC)

Appendix B: Proofs

Lemma

When $\theta = 0$ (no NPEs), there are two equilibria: $(u_1, u_2) = (0, 0)$ and $(u_1, u_2) = \frac{1}{2}(\sqrt{L}, \sqrt{L})$.

Proof. That $(0,0)$ is an equilibrium we can see immediately from the objective function. Suppose we have an equilibrium where $(u_1, u_2) > 0$. Then, dividing the two FOC we have

$$\frac{u_1(V - V_2(0))}{u_2(V_1(2) - V)} = \frac{u_2}{u_1} \Rightarrow u_1 = u_2.$$

Replacing $u_1 = u_2 = u^*$ in the equilibrium condition (FOC 1) we get

$$\frac{u^*(V_1(2) - V)}{4(u_*)^2} = u^* \Rightarrow u^* = \frac{\sqrt{L}}{2}.$$

$\square$

Lemma

When $\theta > 0$ (with NPEs), there are two equilibria: $(u_1, u_2) = (0,0)$ and $(u_1, u_2) > 0$, where $u_2 = Ku_1$ and $K > 1$.

Proof. That $(0,0)$ is an equilibrium we can see immediately from the objective function.

Define $K^2 = \dfrac{L + \theta}{L} > 1$. In an equilibrium where $(u_1, u_2) > 0$ by dividing the two FOC we have

$$\frac{u_1 K^2}{u_2} = \frac{u_2}{u_1} \Rightarrow u_2 = Ku_1.$$

Then, replacing $u_2 = Ku_1$ in the equilibrium condition (FOC 1) we get

$$\frac{KLu_1}{(1+K)^2(u_1)^2} = u_1 \Rightarrow u_1 = \frac{\sqrt[4]{(L+\theta)L}}{1+K}.$$

$\square$

Lemma

The difference $D(\theta) = V^*(\theta) - V^{**}(\theta)$ is positive for all θ and $D(\theta) > D(0)$, for all θ.

Proof.

$$V^*(\theta) = \frac{V_1(2) + KV}{1+K} - c(u_1^*).$$

$$V^{**}(\theta) = \frac{KV + (V_2(0) - \theta)}{1+K} - K^2 c(u_1^*).$$

Then,

$$D(\theta) := V^*(\theta) - V^{**}(\theta) = \frac{2L + \theta}{1+K} + (K^2 - 1)\frac{KL}{2(1+K)^2} > 0.$$

which can be simplified to Replacing the value of K we have:

$$D(\theta) = \frac{(2L + \theta) + \frac{1}{2}(K - 1)KL}{1 + K}.$$

Without the NPE, we have $D(0) = L$. Hence,

$$D(\theta) - D(0) = \frac{3}{2}\frac{\theta L K(K - 1)}{1 + K} > 0$$

$\square$

Lemma

If $V^* \geq V^{**}$ there is a unique symmetric equilibrium where $u_x = u_y = U^*$.

Proof. This problem can be equivalently written as

$$\max_{u_i^x, u_i^y} \frac{(V^{**} - V^*)(u_{-i}^x + u_{-i}^y)}{u_i^x + u_i^y + u_{-i}^x + u_{-i}^y} - c(u_i^x) - c(u_i^y)$$

Thus, as long as $V^{**} < V^*$ the objective function is strictly concave and therefore the first order conditions are necessary and sufficient for an interior solution,

$$\text{FOC w.r.t. } u_i^j : \qquad \frac{(V^* - V^{**})(u_{-i}^x + u_{-i}^y)}{(u_i^x + u_i^y + u_{-i}^x + u_{-i}^y)^2} = c'(u_i^j)$$

Therefore $u_i^x = u_i^y = u_i$. Therefore, we have

$$\frac{2(V^* - V^{**})u_{-i}}{(2u_i + 2u_{-i})^2} = c'(u_i). \quad \text{(FOC)}$$

Using the convexity of $c(\cdot)$ it is easy to show that for any u_{-i} there exists a unique u_i that satisfies the (FOC). Denote the best response function as $u_i(\cdot)$. Equilibrium effort (u_1^*, u_2^*) are given by the solution to

$$u_1(u_2^*) = u_1^*, \quad u_2(u_1^*) = u_2^*.$$

In any equilibrium we must have $u_1^* = u_2^*$. Suppose by contradiction that $u_1^* > u_2^*$. By convexity of $c(\cdot)$ we have

$$\frac{2(V^* - V^{**})u_2^*}{(A^* + 2u_1^*)^2} > \frac{2(V^* - V^{**})u_1^*}{(A^* + 2u_2^*)^2}$$

$$\Leftrightarrow [2(V^* - V^{**})u_2^*](A^* + 2u_2^*)^2 > [2(V^* - V^{**})u_1^*](A^* + 2u_1^*)^2$$

But as long as $V^* - V^{**} > 0$ this implies $u_2^* > u_1^*$, which is a contradiction. $\square$

Appendix C: Bargaining power in the patent trade stage

In this section we show that it is not crucial for the result to assume that the NPE has all the bargaining power when buying patents. In the main text, we assumed that the NPE makes a take-it-or-leave-it offer to the patent owner. In this section, we assume the NPE has bargaining power $b \in [0,1]$ and the patent owner has $(1-b)$.

Notice that bargain power will only be important when there are several alternatives for which the parties can bargain over. In games A, C, and D there is only one alternative that is feasible and there is nothing to bargain over. The only case where the bargain power plays a role is in game B, where there is extra surplus to bargain over, which is extracted from firm 2 by the NPE.

We use the Nash bargaining solution, where firms will bargain over the price p of the patent. If firm 1 and the NPE do not reach an agreement, the payoffs are the payoffs from game 1: $\pi_1^{NA} = \pi_d + \beta(2-\beta)(\pi_d+F)+c$ and $\pi_N^{NA} = 0$. If firms reach an agreement at price p, the payoffs are $\pi_1^A = \pi_d + \beta(\pi_d + F) + c + p$ and $p_N^A = \beta(\pi_d + F) + c - \beta^2 \pi_d - p$. The Nash bargain solution is the price p that solves

$$\max_p (\beta(\pi_d + F) + c - \beta^2 \pi_d - p)^b (p - \beta(1-\beta)(\pi_d + F))^{1-b}$$

Solving this problem we find:

$$p = \beta(\pi_d + F) + c - \beta^2 \pi_d - b(c + \beta^2 F)$$

which implies payoffs:

$$\pi_1^b = \pi_d + \beta(2-\beta)(\pi_d + F) + c + (1-b)(c + \beta^2 F), \quad \pi_N^b = b(c + \beta^2 F).$$

This is, the NPE obtains exactly a fraction b of the extra surplus that is generated by buying firm 1's patent.

Therefore, the only change in our results will come in the effort choice in Lemmas 2 to 4, althought the qualitative results are the same. Let $B = (1-b)(c + \beta^2 F)$. In Lemma 2, the only change is that $K^2 = \frac{L+\theta}{L+B}$, which is still greater than 1, and equal to 1 when $b = 0$. In equilibrium $u_2 = Ku_1$ and $u_1 + u_2 = \sqrt[4]{(L+\theta)(L+B)}$. Using this result, we can generalize Lemmas 3 and 4:

$$D(\theta) = \frac{(2L + B + \theta) + \frac{1}{2}(K-1)K(L+B)}{1 + K}.$$

Without the NPE we have $D(0) = L$. Hence, since $B > 0$,

$$D(\theta) - D(0) > \frac{3}{2} \frac{\theta L K(K-1)}{1 + K} > 0.$$

Appendix D: to section 7.1

We index the licenses offers as (L^x, L_2^y, L_3^y) and the accept/reject decisions accordingly.

- (A,A,A): $\pi_1 = \pi_T + L_2^y + L_3^y$, $\pi_2 = \pi_T - L_2^y - L^x$, $\pi_3 = \pi_T - L_3^y + L^x$.
- (A,A,R): $\pi_1 = K^+ + \beta I(\pi_d - \pi_T) + L_2^y$, $\pi_2 = \pi_T - L_2^y - L^x + \beta I(\pi_d - \pi_T)$, $\pi_3 = K^- + L^x - \beta I\pi_T$.
- (A,R,A): $\pi_1 = K^+ + \beta I(\pi_d - \pi_T) + L_3^y$, $\pi_2 = K^- - \beta I\pi_T - L^x$, $\pi_3 = \pi_T - L_3^y + L^x + \beta I(\pi_d - \pi_T)$.
- (R,A,A): $\pi_1 = \pi_T + L_2^y + L_3^y + \beta I(\pi_d - \pi_T)$, $\pi_2 = K^- - \beta I\pi_T - L_2^y$, $\pi_3 = K^+ - L_3^y + \beta I(\pi_d - \pi_T)$.
- (A,R,R): $\pi_1 = K^+ + \beta(F + R) - c + (\beta I)^2(\pi_M - 2\pi_d + \pi_T) + 2\beta I(\pi_d - \pi_T)$, $\pi_2 = K^- + \beta I(1 - \beta I)(\pi_d - \pi_T) - \beta I\pi_T - L^x$, $\pi_3 = K^- + \beta I(1 - \beta I)(\pi_d - \pi_T) - \beta I\pi_T + L^x$.
- (R,A,R): $\pi_1 = K^+ + (\beta I)^2(\pi_M - 2\pi_d + \pi_T) + 2\beta I(\pi_d - \pi_T) + L_2^y$, $\pi_2 = K^- + \beta I(1 - \beta I)(\pi_d - \pi_T) - \beta I\pi_T - L_2^y$, $\pi_3 = K^- + \beta I(1 - \beta I)(\pi_d - \pi_T) - \beta I\pi_T + \beta(F + R) - c$.
- (R,R,A): $\pi_1 = K^+ + \beta I(2 - \beta I)(\pi_d - \pi_T) + L_3^y$, $\pi_2 = K^- - \beta I(2 - \beta I)\pi_T - \beta(F + R) - c$, $\pi_3 = K^+ + \beta I(2 - \beta I)(\pi_d - \pi_T) - L_3^y$.
- (R,R,R): $\pi_1 = K^+ + \beta(F + R) - c + (\beta I)^2(2 - \beta I)(\pi_M - \pi_d) + \beta I[3(1 - \beta I) - (\beta I)^2](\pi_d - \pi_T)$, $\pi_2 = K^- - \beta(F + R) - c + \beta I(1 - \beta I)^2(\pi_d - \pi_T) - \beta I(2 - \beta I)\pi_T$, $\pi_3 = \pi_T + [1 - (1 - \beta I)^3](\pi_d - \pi_T) - \beta I\pi_d - 2c$.