

City and Suburban Competition

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Abstract

This paper presents a model of the local public sector with a city and a suburb, whose mayors engage in Cournot-duopoly-type competition. Each mayor can extract resources from residents, although residents's ability to move and the other mayor's choices limit this extraction. The suburban mayor's reaction function and the Nash equilibria are more complicated than in a standard Cournot model because land rents, which have no analogue in a Cournot model, affect the mayors' incentives. Some simulation results resemble those of an Alonso model. Comparative statics results are derived for changes in agricultural land prices and transportation costs and their effect on city population and rents captured by the city mayor.

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1 Introduction

1.1 Imperfect Competition and Local Public Finance

Strategic models that changed how economists think about competition among firms have had less effect on how economists think about competition among towns and cities. Three non-strategic models dominate local public finance. In Tiebout models individuals choose among a set of small jurisdictions offering different tax and public service bundles and towns that are deemed to compete like price-taking firms in a large market. In "Leviathan" models residents typically lack an exit option and cannot directly prevent politicians from exerting monopolist powers. Most Alonso and von Thünen-type land-use models typically omit any description of political processes.¹ Just as perfect competition and monopoly fail to describe strategic interactions in product markets, typical Tiebout and Leviathan models fail to capture strategic interactions among cities and suburbs because in Tiebout models cities take prices as given, so actions of rivals matter only in an aggregate implicit manner, while in a usual Leviathan model no rivals exist.²

This paper presents an analogue to a Cournot duopoly in a model of the local public sector with one city and one suburb. The model focuses on the behavior of two mayors who can extract resources from residents. Individuals "vote with their feet," but cannot choose their mayor. Nor can they punish a mayor except by moving to the other jurisdiction. This describes a situation in which candidates do not vary in competence or ideology, cannot make binding promises to voters and can only serve one term. Here mobility constrains governments more than politics. Extending the model to include politics, multiple suburbs and other features is left to future research.

Translating the Cournot duopoly model into the context of local public economics is more than a matter of renaming variables because land rents play an important role. As residents move from one jurisdiction to another, they impose a pecuniary externality on others through changes in land prices. Land rents also affect opportunities available to the city mayor, who by charging taxes higher than costs of production, may be able to extract some land rents.

Epple and Zelenitz (1981) developed a model a Leviathan city surrounded by a competitive fringe of suburbs. Individuals consume a private good and housing that is made via a convex technology. Perturbations around a Nash equilibrium are used to investigate strategic interactions between a surplus-maximizing city mayor and suburban mayor. The

¹Konishi (1996) is an exception to this, providing a model in which residents affect outcomes both through voting and migration.

²Not all tax competition papers take this approach. Eggert (1999) uses a Leviathan setup to analyse tax competition and the benefits of international tax coordination.

city mayor, in Nash equilibrium, can extract some rents because land is nonreproducible.

The central focus here is the mutual determination of fiscal decisions in cities and suburbs. If the city extracts land rents, then a self-interested suburban mayor can also capture rents, to the extent she is unconstrained by the political process. Thus, if agglomeration economies or special amenities of city living generate land rents, then the city can charge higher taxes for a given service bundle. If the city's taxes exceed the minimal cost of provision, however, then a suburban mayor may also impose taxes in excess of minimum cost requirements. Thus, competition in taxes and services to attract and retain residents links city and suburban fiscal behavior. Competition will also limit the ability of either mayor to extract land rents from residents stemming from the city's locational advantage. How land rents are split between residents and mayors will depend on transportation costs and the value of suburban land in non-housing uses, such as agriculture.

Including land and the features of a simple spatial model complicates the model. Choosing a suitable functional form is often necessary to generate closed-form reaction functions. For this reason the model employs quasi-linear preferences with a quadratic subutility function for housing. Quasi-linear utility functions are commonly used in the political economy literature (e.g. Persson, Roland and Tabellini 2000). Models with specific functional forms shed light on what comparative statics results might go through in a general setting. Just as Cournot-Nash and Bertrand-Nash models with linear demands help show how firms interact strategically, the quasi-linear functional forms used here help illustrate how local governments interact.

1.2 Competition Among Governments

One can draw parallels between individuals who choose where to live and whom to vote for with consumers who choose what products to buy, and between public officials and entrepreneurs. City officials succeed by pleasing residents enough to vote for them, rather than for others, or live in their cities, instead of somewhere else. City officials, like entrepreneurs, benefit by assembling an attractive bundle of goods and services and by charging a competitive price or tax. As outside options constrain entrepreneurs to set prices closer to marginal costs (and to the extent that some long-run adjustment process brings marginal costs towards average costs), competition promotes efficiency.

This is, of course, Tiebout's celebrated insight: perfect competition among many governments might let each consumer pick a bundle of publicly provided goods and services that matches her preferences and a tax that reflects efficient provision costs. Thus, jurisdiction in the midst of many others have limited ability to exploit their residents.³ A natural in-

³For an unsympathetic analysis of the Tiebout model, see Bewley (1981).

terpretation of Tiebout's model is a large collection of suburbs, in which the choices of any one suburb have little effect on the whole.⁴

Some Tiebout-like models incorporate some features of imperfect competition. Epple and Zelenitz (1981) find that individual mobility of residents within a set of competing jurisdictions cannot prevent a bureaucratic monopolist from capturing rents. This result occurs because the stock of land and jurisdictional boundaries are fixed. Residents need other ways to constrain politicians, such as electoral accountability, to achieve efficient governance. A second generation of fiscal federalism research looks at how jurisdictions compete with each other when factors of production are mobile. Wildasin (1988) considers a collection of jurisdictions engaged in fiscal competition, each with a single individual, and all competing for mobile capital. He finds the Nash equilibria for a game in which jurisdictions compete by setting tax rates differ from the Nash equilibria for a game in which jurisdictions compete in public expenditure levels.⁵ Wildasin (2003) presents a dynamic tax competition game in which factors of production are mobile, but are subject to adjustment costs. This gives governments an incentive to tax relatively immobile factors more heavily, despite the long-term costs of such a policy.

Policy-oriented researchers have also become increasingly interested in competition among governments. Breton's (1996) analyzes "internal competition" and "external competition" among a many different types of governments. Brandl (1998) argues that injecting more competition among U.S. state governments operations would yield higher levels of efficiency and responsiveness.

In the model presented here, residents live in either a city or a suburb. Both have unelected mayors who may attempt to extract rents. While residents cannot choose who run the government, they can vote with their feet by moving to another jurisdiction. If more voters move from the city to the suburb, the suburb expands into the surrounding agricultural area, so the supply of suburban housing is perfectly elastic. While the ability of developers to convert "raw land" into housing tethers the suburban housing price to the value of agricultural land, the suburban mayor may extract more rents if demand for suburban location is high. City and suburb are then intertwined by migration decisions, so that the population and efficiency of government are determined through a nontrivial strategic interaction between the city and suburban mayors.

⁴For an empirical application of the Tiebout model see Epple, Romer and Sieg (2001).

⁵This appears similar to the results of Vives (1985) and Vives and Singh (1986) for Bertrand and Cournot competition with differentiated products.

1.3 The Model

The model comprises a closed metropolitan area of one city C and a suburb S . The metropolitan area contains a continuum $[0,1]$ of residents. The city's population with N_C residents and a mayor. The suburb has $N_s = 1 - N_C$ residents and its own mayor. All residents must work in a central business district, and getting there costs city residents zero and suburban residents t . More generally, t represents locational advantages of the city that do not depend on population size. Thus t might represent agglomeration effects or special urban amenities unavailable in the suburbs.

The city's boundaries are fixed and contain one unit of land, but the suburb's boundaries are not fixed. Agricultural land, which yields rent p_A per unit, can be converted to residential use in the suburb at zero cost. The suburban government cannot restrict the supply of land for residential use. The cost of structures is ignored, so ground rent is the sole cost of housing.

Each resident earns a fixed income ω and consumes a private consumption good c , which serves as the numeraire, and housing s . Housing costs p_C in the city. The supply of suburban land is perfectly elastic at price p_A . One half of the metropolitan area are endowed with an equal share of the single unit of city land, and the other half of residents own no land. Other patterns of land ownership could just as easily be introduced. Let $l_C(i)$ represent the endowment of land for person i , defined as

$$l_C(i) = \begin{cases} 2 & \text{if } i \in [0, \frac{1}{2}] \\ 0 & \text{if } i \in (\frac{1}{2}, 1] \end{cases}$$

where the city land adding-up constraint is then $\int_0^{1/2} l_C(i) \cdot di = 1$. These land endowments could be omitted without changing any modelling results.

Agricultural land, which could be used for housing in the suburb, is owned by farmers outside of the metropolitan area. While this implies the model not fully closed, this matters little because these farmers earn no rents.⁶

Imperfect competition in the political arena lets politicians capture land rents that can be diverted to either maintain political power or towards increasing redistributive expenditures. Of course, these aims can have both political and altruistic interpretations. For example, the mayor might run a patronage regime in which jobs, in which low productivity is tolerated, are exchanged for political support; or perhaps the mayor has a stronger taste for redistribution than a typical resident.⁷ The city mayor can charge city residents γ_C and the suburban mayor

⁶The model could endow some residents with agricultural land while reducing their fixed income. This modified model would be fully closed, but only would introduce additional notational complication.

⁷Few mayors who headed patronage machines earned large personal fortunes. Some, such as James Curley

can charge residents γ_S . The quantities γ_C and γ_S may represent additions to a mayor's personal or political "slush fund" or redistributive expenditures which indirectly benefit the mayor but do not enter the utility function of residents. The city mayor's objective is to maximize $N_C \cdot \gamma_C$, and the suburban mayor maximizes $N_S \cdot \gamma_S$. The city provides a local public good g_C financed by tax τ_C , and the suburb provides g_S and charges proportional tax τ_S . Each government uses a linear production technology. In the suburb one unit of tax revenues produces f units of per capita public good provision, so $g_S = f \cdot \tau_S$.

1.4 Metropolitan Residents' Behavior

All residents have a quasi-linear utility function $u(c, s) = c + \sqrt{s} + k \ln g$. City residents face a budget constraint

$$\omega + [l_C(i) - s] \cdot p_C - c - \gamma_C - \tau_C \geq 0,$$

and suburban residents face a budget constraint

$$\omega + l_C(i) \cdot p_C - s p_S - c - t_S - \gamma_S - \tau_S \geq 0.$$

Residents act nonstrategically and the mayors anticipate the behavioral responses of residents. Thus, residents take prices and the actions of mayors as given. Residents choose the tax rate in their jurisdiction by voting. A suburban resident's maximization problem (*UtilMax-S*) and a city resident's problem (*UtilMax-C*) are then

$\max u(c, s) = c + \sqrt{s} + k \ln g$ such that

- (i) $\omega + p_C \cdot l_C(i) + p_A \cdot s - c - t - \gamma_S - \tau_S \geq 0$ (λ) (*UtilMax-S*)
- (ii) $c \geq 0$; (η)
- (iii) $s \geq 0$ (μ)

$\max u(c, s) = c + \sqrt{s} + k \ln g$ such that

- (i) $\omega + p_C \cdot l_C(i) - p_C \cdot s - c - \gamma_C - \tau_C \geq 0$ (λ) (*UtilMax-C*)
- (ii) $c \geq 0$;
- (iii) $s \geq 0$.

For interior solutions the demand functions are:

of Boston, died broke, suggesting either a strong interest in redistribution and sharing of gains to cultivate political power and influence, or a limited ability to manage money.

$$s(p_C) = 1/4p_C^2 \quad \text{individual housing demand in the city}$$

$$s(p_S) = 1/4p_A^2 \quad \text{individual housing demand in the suburb}$$

$$c(\omega, p_C; \gamma_C) = \omega + p_C \cdot l_C(i) - 1/4p_C - \gamma_C - \tau_C \quad \text{private consumption in the city}$$

$$c(\omega, p_A, t; \gamma_S) = \omega + p_C \cdot l_C(i) - 1/4p_A - t - \gamma_S - \tau_S \quad \text{private consumption in the suburb.}$$

Voters choose $\tau_j^* = k$, $j = \{S, C\}$ unanimously in each jurisdiction. In the city the land market clears when $N_C = 4p_C^2$. Because the city population cannot exceed one the price of city housing must be less than one half for the city land market to clear. Thus we restrict attention to situations where $1/2 \geq p_C$.⁸

The equilibrium rental rate for city housing is $p_C^* = \frac{1}{2}\sqrt{N_C}$, so that prices for city housing rise with city population. Similarly, total suburban demand for housing is $4 \cdot (1 - N_C) \cdot p_A^{-2}$, while the supply of housing is perfectly elastic at price p_A .

In equilibrium residents receive equal utility in each jurisdiction because they are mobile.⁹ Indirect utility for a city resident at the equilibrium housing price is

$$v(p_C, \gamma_C; \omega) \Big|_{p_C = \frac{\sqrt{N_C}}{2}, \tau_j^* = k} = \omega + \frac{1}{2}l_C(i) \cdot \sqrt{N_C} - k + \frac{1}{2\sqrt{N_C}} - \gamma_C + k \ln(f \cdot k).$$

Indirect utility for a suburban resident at the equilibrium housing price is

$$v(p_A, \gamma_S, t; \omega) \Big|_{p_C = \frac{\sqrt{N_C}}{2}, \tau_j^* = k} = \omega + \frac{1}{2}l_C(i) \cdot \sqrt{N_C} - k + 1/4p_A - t - \gamma_S + k \ln(f \cdot k).$$

The equal utility condition, equating the indirect utility functions of a city and a suburban resident, for owners and non-owners alike is then:

$$\frac{1}{2\sqrt{N_C}} = \frac{1}{4p_A} + (\gamma_C - \gamma_S) - t.$$

Solving for N_C gives the city's population supply function, which is similar to functions defined by Fujita (1989, pp.140-50).

⁸The price of housing in the city is not an economic fundamental. How this upper bound on the city housing price relates to economic fundamentals will be discussed below in the section on capitalization. That section shows there is an increasing monotonic relationship between the agricultural rental rate and the city housing price for a given level of transportation costs.

⁹This presumes that all residents have interior solutions to maximization programs in both locations. If sorting among residents with different levels of endowment occurs, some may have strictly greater indirect utility in one locations than in another. This possibility is considered in more detail in the next section.

$$N_C(\gamma_C, \gamma_S; t, \varepsilon) = \min \left[1, \left(\frac{2p_A}{1 + 4p_A[\gamma_C - \gamma_S - t]} \right)^2 \right].$$

Population depends on three things: the difference in slack $\{\gamma_C - \gamma_S\}$, determined by the strategic interaction between the city and suburban mayors; the price of agricultural land p_A ; and the city's economic advantages reflected in the transportation cost t .

All citizens live in the city if $\gamma_S - \gamma_C + t \geq 1/4p_A - \frac{1}{2}$. Thus, the suburb only exists if the opposite inequality holds, that is, when suburban housing is cheap enough to offset the city's natural economic advantages and the difference in levels of slack. If the endowments are too small to allow residents positive levels of consumption, residents must reduce either public or housing expenditures.¹⁰ City population approaches zero as agricultural land prices approach zero, which is an economically uninteresting case. The suburban population approaches zero as transportation or productivity differences make locations outside the city increasingly unattractive relative to city locations. This case is examined in more detail in Appendix 1. The remainder of this paper focuses on situations in which all residents find interior solutions to their maximization programs.

1.5 City and Suburban Mayors' Behavior

Each mayor decides simultaneously what level of surplus to extract from their residents. Raising γ_C increases the resources taken from each city resident, but decreases city population. Higher transportation costs (t) or higher agricultural rents (p_A), however, increase the city's population, *ceteris paribus*. The suburban mayor faces a similar decision, although these decisions are asymmetric, however, because of the city's locational advantage.

The population supply function acts like a demand for city residence function and γ_C acts like a price variable controlled by the mayor. The revenue captured by the city mayor as a function of γ_C resembles a total revenue curve from a standard monopoly pricing problem. The city mayor can then maximize the resources at her disposal by finding the highest point of a "total slack" function. Figure 1 presents a family of "total slack" functions.

¹⁰In a richer model that includes different endowments for different people, analysis of corner solutions becomes important because some voters would have zero consumption if they changed towns. If only some voters have interior solutions to maximization programs in both towns then the elasticity of population with respect to slack decreases *ceteris paribus* relative to a situation where all voters had interior solutions.

Figure 1: A Family of Total Slack Functions for the City Mayor

Legend: From the lowest to highest lines: ; $t = 0.5$; $t = 0.6$; $t = 0.7$. For all lines shown here $p_A = 0.2$; $\gamma_S = 0.2$; $\varepsilon = 0$

The city mayor solves the program

$$\max_{\gamma_C \geq 0} \gamma_C \cdot N_C(\gamma_S, \gamma_C; t). \quad (Slack-C)$$

The first-order condition rearranges to yield

$$\frac{\partial N_C(\gamma_S, \gamma_C; \cdot)}{\partial \gamma_C} \frac{\gamma_C}{N_C(\gamma_S, \gamma_C; \cdot)} = -1,$$

which resembles the familiar monopolist's condition: at the point generating maximum revenue the elasticity of city population with respect to γ_C equals minus one. Inserting the population supply function gives the city mayor's reaction function:

$$\gamma_C(\gamma_S; p_A, t) = \max(1/4p_A - \gamma_S - t, 0),$$

which is depicted in Figure 2 for three sets of parameter values. The form of the city mayor's reaction function implies two results.

Proposition 1 If all residents face interior solutions in the city and suburb, the combined increases in city and suburban slack will completely offset a decrease in transportation costs.

Proof: Obvious, given that for any Nash equilibria the city mayor uses his best response function.

Proposition 1 highlights differences between the Cournot-Nash equilibrium for firms in a product market and the equilibrium in the city-suburb game. Reductions in transport costs are not passed on to residents. However, having suburban competition does prevent the city mayor from expropriating all available wealth from residents. Without this competitive constraint private and public consumption would be zero. Proposition 1 does not state whether it is city or suburban slack which rises to offset decreases in transport costs. How each mayor's rate of slack varies in equilibrium depends on the elasticity of population with respect to city and suburban slack.

Before analyzing the suburban mayor's behavior, Proposition 2 describes a lower bound on the city rate of slack. This result follows from the quadratic form of the housing subutility function, and this threshold would certainly change for another specification.

Proposition 2 If all residents face interior solutions in the city and suburb and if the city mayor is a total slack maximizer, then the city rate of slack is at least one-fourth: $\gamma_C \geq \frac{1}{4}$.

Proof: Suppose $\gamma_C > 0$. Substituting the reaction function into the population supply function gives result $N_C = \frac{2p_A}{8p_A\gamma_C} = 1/4\gamma_C$. If $\gamma_C = 0$ then $1/4p_A - \gamma_S - t \geq 0$ which implies the denominator term of the population supply function is $1 + 4p_A[\gamma_C - \gamma_S - t]$ is either zero and city population is undefined, or negative which describes an economically irrelevant case in which increasing city slack leads to a higher city population so $\gamma_C = 0$ cannot be a maximizing value.

Higher transportation costs shift the city mayor's reaction function downwards, and higher agricultural rents shifts the reaction function upwards. Figure 2 shows the city mayor's reaction function for selected parameter values.

Figure 2: City Mayor's Reaction Function

Legend: $p_A = 1/2$ for dashed line; $p_A = 1/3$ for dot-dashed line; and $p_A = 1/4$ for the solid line; $t = 1/8$ for all lines.

The suburban mayor's problem is similar, but not symmetric due to the suburb's elastic border and transportation costs. The suburban mayor solves the following problem:

$$\max_{\gamma_S \geq 0} [1 - N_C(\gamma_C, \gamma_S; t)] \cdot \gamma_S \quad (\text{Slack-}S)$$

which again leads to the familiar monopoly pricing rule for the first-order condition. Substituting the population supply function into the first-order condition and solving yields the suburban mayor's reaction function

$$\gamma_S(\gamma_C; p_A, t) = \max \left(\frac{Z(\gamma_C; p_A, t)}{2} - \frac{1}{6 \cdot Z(\gamma_C, p_A; t)} + \frac{1}{4p_A} + (\gamma_C - t), 0 \right),$$

where

$$Z(\gamma_C; p_A, t) \equiv \sqrt[3]{\left(\sqrt{\frac{1}{27} + \left(\frac{1}{p_A} + \gamma_C - t \right)^2} + 3(\gamma_C - t)^2 - 2 \left[\frac{1}{4p_A} + (\gamma_C - t) \right] \right)}.$$

The suburban mayor's reaction function is shown in Figure 3 for three levels of agricultural rent. As agricultural rent falls, the suburban reaction function shifts upwards. As

before, the suburban mayor's reaction function is restricted to positive values.¹¹

Figure 3: Suburban Mayor's Reaction Function

Legend: $p_A = 1/2$ for solid line; $p_A = 1/3$ for dot-dashed line; and $p_A = 1/4$ for the dotted line; $t = 1/8$ for all lines. Vertical axis represents the suburb's rate of slack.

2 Nash Equilibrium

A Nash equilibrium is a strategy pair $\{\gamma_C^*, \gamma_S^*\}$ such that both mayors use their best response functions. Unlike the Cournot duopoly model with linear demand, which generates simple closed-form solutions, finding closed-form reaction functions is impossible here. Comparative statics for Nash equilibria, however, can be illustrated with a set of graphs using a range of key the parameter values: the level of agricultural rent and the city's economic advantage t .

¹¹For finite parameters the suburban population cannot reach one, and the maximization is irrelevant if the suburban population is zero. Thus boundary solutions are impossible here.

2.1 Determination of Equilibria: Simulation Results

In Nash equilibrium the rates of slack chosen by the mayors are a function of agricultural rents and transportation costs. Figure 4 shows Nash equilibria for three different levels of agricultural land rent, while the difference in transportation costs is held at $1/8$. In Figure 4, reaction functions are shown for agricultural rent levels of $1/3$, $1/4$ and $1/5$. The city mayor's reaction function is affine while the suburban mayor's reaction function is slightly nonlinear. As agricultural rents rise, the Nash equilibrium moves to the "southeast", as city slack increases and suburban rent decreases as the suburb becomes a relatively less attractive place to live.

Figure 4: Nash Equilibria for Various Levels of Agricultural Rent

Legend: From bottom to top: $p_A = 1/2$ for solid lines; $p_A = 1/3$ for dashed lines; and $p_A = 1/4$ for the dotted lines; $p_A = 1/5$ for the dot-dashed lines; $t = 1/8$ for all lines. Vertical axis represents the suburb's rate of slack.

Next transportation costs are varied while agricultural rent is held fixed. In Figure 5, reaction functions are shown for transport costs of $1/8$, $1/4$, $3/8$, $1/2$ and $5/8$. Higher transportation costs shift the city mayor's reaction function to the right, and shift the suburban mayor's reaction function down. Thus in Nash equilibrium, city slack increases and suburban rent decreases as transportation costs rise, so the locus of equilibria moves "southwest"

as t increases. Thus increasing agricultural rents and transportation costs have roughly opposite comparative statics effects. Proposition 1 implies that a decrease in transportation costs will be offset by the combined increased rates of slack for the city and suburb. This result can be seen from the Figure 5. As transportation costs fall from $t=5/8$ to $t=1/8$, city slack increases from 0.2862 to 0.3796, an increase of 0.0933, while suburban slack increases from 0.0888 to 0.4954, an increase of 0.4066. Suburban slack absorbs in this case over 80% of the increase in transport costs.

Figure 5: Nash Equilibria for Various Transportation Costs

Legend: From top to bottom: $t = 1/8$ for dotted lines; $t = 1/4$ for dashed lines; $t = 3/8$ for solid lines; $t = 1/2$ for dot-dashed lines; $t = 5/8$ for the lowest set of lines; $p_A = 1/4$ for all cases.

2.1.1 Rates of Slack in Equilibrium

Equilibrium rates of slack depend on transport costs and the value of agricultural land. Figure 6 illustrates the relationship between agricultural rent and the city rate of slack. Higher agricultural land values reduce the city rate of slack, and higher transportation costs shift the schedule of city slack downwards. With transport costs equal to $1/8$, city slack approaches its maximum of two as agricultural land value approaches zero. When transport

costs equal $1/4$, city slack approaches its maximum of one as agricultural rent approaches zero. As agricultural land values approach zero however, the city population also approaches zero, so the city mayor collects higher rates of slack from a smaller and smaller population. An economically meaningless solution runs along the horizontal axis.

Figure 6: Equilibrium City Rate of Slack as Function of Agr. Rent

Legend: Solid line shows city rate of slack in Nash equilibrium when $t = 1/8$; dotted line shows suburban rate of slack when $t = 1/8$; dashed line shows $t = 1/4$. Vertical axis represents the city rate of slack.

Figure 7 shows the equilibrium city rate of slack as a function of transport costs. Increasing agricultural land value shifts the locus of equilibrium rates of slack inwards and makes the curve steeper. As Proposition 2 implies, the city rate of slack is negatively related to transportation costs. As Figure 7 shows, the slope becomes more steeply negative at higher levels of transportation costs, which is where the suburban population shrinks towards zero.

Figure 7: Equilibrium City Rate of Slack as Function of Transport Costs

Legend: Solid line shows $p_A = 1/4$; dashed line shows $p_A = 1/3$; vertical axis shows rate of city slack.

2.2 Determinants of City Population

The Nash equilibrium city population is a function of agricultural rent and transport cost. Figure 8 shows the city population function for various levels of transportation costs. As the price of agricultural land rises, implying that suburban housing becomes more expensive, the city population rises until everyone lives in the city. Increased transportation costs shift the city population function upwards. If the value of agricultural land is zero, then no one lives in the city.

Figure 8: City Population as a Function of Agricultural Rent

Legend: Solid line shows the population function for $t = \frac{3}{10}$; dashed line shows $t = 1/4$; dotted line shows $t = 1/5$.

In Figure 9 the city population is shown as a function of transportation costs with the agricultural rent held fixed at two. As expected, city population increases as transportation costs rise. An increase in the cost of agricultural land also increases city population.

Figure 9: City Population as Function of Transport Cost

Legend: Solid line shows $p_A = 1/3$; dashed line shows $p_A = 1/4$; dotted line shows $p_A = 1/5$.

2.3 Capitalization Effects

As higher rates of slack reduce the benefits city residents enjoy, rental rates for city land will decline according the usual theory of capitalization. The underlying economic parameters then affect city rental rates through intermediate channels of slack and housing demand. The city housing price as a function of the mayors' strategic variables is then

$$p_C^*(\gamma_C, \gamma_S; p_A, t) = \frac{1}{2} \sqrt{\left[4 \left(\frac{p_A}{4p_A(\gamma_C - \gamma_S - t) + 1} \right)^2 \right]} = \left(\frac{p_A}{4p_A(\gamma_C - \gamma_S - t) + 1} \right)$$

The excess of city slack over suburban slack (where both are nonnegative) is

$$\gamma_C - \gamma_S = \frac{6}{7}t - \frac{3}{7} - \frac{1}{14} \sqrt{(4t^2 - 4t + 29)}$$

so the equilibrium rental rate for city housing is

$$p_C^*(p_A, t) = \left[\frac{1}{p_A} - \frac{4}{7} \left(t + 3 + \frac{1}{2} \sqrt{4t^2 - 4t + 29} \right) \right]^{-1}.$$

The city rental function's dependence on transport costs is shown in Figure 10. The lower lines show the agricultural rental rate and the higher lines show the price of city housing. For example, the lower-lying solid line shows $p_A = 1/4$ and the higher-lying rental rate shows the corresponding city housing rate when $p_A = 1/4$. As transportation costs rise the city rental rate rises. At lower levels of p_A higher transportation costs have little effect on city housing prices. In particular, the city price function's slope is less than 1, implying incomplete capitalization of the city locational advantages in property prices.

Figure 10: Equilibrium Price of City Housing as a Function of Transportation Costs

Legend: The solid line drawn for city rental rate when $p_A = 1/4$; dashed line when $p_A = 1/5$; dotted line when $p_A = 1/6$;

Figure 11 illustrates the agricultural rental rate's effect on the price of city housing. When the price of agricultural land rises, city and suburban housing becomes more expensive. Higher transportation costs shift city housing costs upwards, but only slightly. Doubling the transportation cost from $1/4$ to $1/2$ has only slightly affects city housing prices.

Figure 11: Equilibrium Price of City Housing as a Function of Agricultural Rental Rate

Legend: The solid line shows $t = \frac{1}{4}$; dotted line shows $t = 1/2$;

3 Discussion and Conclusion

While the Cournot model with two competing firms provides straightforward results, the analogous model for a competing city and suburb has both some familiar aspects and some surprises. As in standard monopoly and Cournot-Nash models, revenue maximization requires a unit elasticity rule. The strategic interaction of city and suburban politicians, which generates competition that lets residents retain much of the land rents, is more complicated than how Cournot oligopolists interact. The pecuniary externality caused by residents moving from one jurisdiction to another has no parallel in Cournot-Nash models.

Some comparative statics results resemble those of non-strategic Alonso-von Thunen spatial models, confirming that housing prices in this model obey well-established principles. For example, city housing prices rise with both transportation costs and the agricultural rental rate. On the other hand, some differences exist. For example, in this model higher levels of slack offset decreased transportation costs. This result raises an important issue for transportation policy: suburban governments may reap most of the benefits generated

by government subsidies for transportation investments intended to lower commuting costs, unless suburban residents can impose political or electoral constraints.

The model also highlights the fiscal interdependence of cities and their suburbs. Suburban competition acts to reduce slack and increase efficiency in the city's production of public goods valued by the median voter. On the other hand, cities with less efficient public production will give suburban politicians the opportunity to spend money on projects not highly valued by suburban voters, to the extent those politicians are unconstrained by political mechanisms.

In such a simple setup, there are many obvious extensions. Having multiple suburbs would provide a different competitive environment, as each suburban mayor would compete with other suburban mayors having similar locational characteristics. Heterogeneity among suburbs would create an interaction between the rent gradient and strategic behavior. Heterogeneity among individuals would lead to sorting of residents by type, potentially allowing the city or suburbs to capture some differential rents. Introducing a convex housing supply function may change the strategic interaction between the city and its suburb by letting the price of housing differ from the price of raw land.

Finally, this positive model may provide a first step in building models that inform urban policy. This model, with suitable extensions to allow calibration with parameters drawn from empirical studies, could serve as a basis for estimates of the benefits of governmental competition. Much of traditional urban policy assumes that consolidating jurisdictions will lead to lower costs of public provision as agglomerated jurisdictions capture scale economies and geographically larger jurisdictions may also be more effective in redistributive activities. Breton (1996) provides a detailed overview of these issues. The potential advantage of combining smaller jurisdictions into larger jurisdictions, or from not breaking up larger jurisdictions into more localized pieces will be at least partially offset by the effects of decreased governmental competition. Analyzing these trade-offs may help higher-level governments provide better quality public services to their citizens.

4 References

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Appendix 1: Corner Solutions

If the endowments are small enough relative to taxes and housing prices residents will be forced to a corner solution.¹² City population approaches zero as agricultural land prices approach zero, which is an economically uninteresting case. The suburban population approaches zero as transportation or productivity differences make locations outside the city increasingly unattractive relative to city locations. This case is examined in more detail here. If the Lagrange multiplier $\eta > 0$ for the maximization program *UtilMax-S* then private consumption is zero and residents reduce housing expenditures, and if residents at a corner solution include the decisive suburban voter, the level of public provision will also be reduced. Housing expenditure is

$$s = \frac{\omega + p_C \cdot l_C(i) - t - \tau_S - \gamma_S}{p_A} \leq \frac{1}{4p_A^2}$$

Consider a comparative statics exercise where transportation costs increase. As these costs rise, the half of the population which does not own property will be forced to a corner solution first, while property owners will initially have interior solution. Non-property residents will then leave the suburb in greater numbers, so the decisive voter will be a property owner, who will set the tax rate $\tau_S^* = k$. However, once property owners are at a corner solution the tax rate falls to $\tau_S^* = 2kp_A\sqrt{s}$, so

$$s = -k + \sqrt{k^2 + \frac{\omega + p_C \cdot l_C(i) - t - \gamma_S}{p_A}} \quad \text{and} \quad \tau_S^* = k \cdot 2p_A.$$

This alters the equal utility conditions, and thus the elasticity of population with respect to slack. Indirect utility for residents in the suburb at a corner solution will be

$$v^{CORNER}(p_A, \gamma_S, t; \omega) \Big|_{p_C = \frac{\sqrt{N_C}}{2}, \tau_j^* = k} = \sqrt{l_C(i) + \frac{\omega - t - \tau_S - \gamma_S}{p_A}} + k \ln(f \cdot \tau_S^*)$$

If owners can solve their maximization problem with an interior solution in both the city and suburb then non-owners will strictly prefer to live in the city if would face a corner solution they resided in the suburb. This result is Proposition 1, which relies on a single-crossing property of preferences. Because the voter on the margin between living in the city

¹²In a richer model that includes different endowments for different people, analysis of corner solutions becomes important because some voters would have zero consumption if they changed towns. If only some voters have interior solutions to maximization programs in both towns then the elasticity of population with respect to slack decreases ceteris paribus relative to a situation where all voters had interior solutions.

and in the suburb is an owner, and because owners have the same equal utility condition as in the above section, the population supply function is not affected by nonowners facing a corner solution.

Proposition A.1 If owners can solve their maximization problem with an interior solution in both the city and suburb and if non-owners would be consumption constrained if they resided in the suburb but not in the city, then non-owners choose to live in the city.

Proof: If the consumption constraint is strictly binding for nonowners and owners are decisive then

$$s = (\omega - t - k - \gamma_S) / p_A < 1/4p_A^2$$

so (i) $1/2p_A / > \sqrt{\frac{\omega - t - k - \gamma_S}{p_A}}$ and (ii) $\frac{1}{4p_A} > \omega - t - k - \gamma_S$. The equal utility condition for owners is

$$\frac{1}{4p_A} = \frac{1}{2\sqrt{N_C}} + \gamma_S - \gamma_C + t.$$

An lemma is now needed to show a key inequality holds.

Lemma: If the consumption constraint is strictly binding for nonowners then

$$\frac{1}{4p_A} > \sqrt{(\omega - t - k - \gamma_S) / p_A} - (\omega - t - k - \gamma_S).$$

Proof: Let $\omega - t - k - \gamma_S = W$ and $p_A = d > 0$ and note $W \geq 0$. Then the inequality becomes:

$$\begin{aligned} \frac{1}{4d} + W &> \sqrt{\frac{W}{d}} \\ \left(\frac{1}{4d} + W\right)^2 &= \frac{1}{16d^2} + \frac{1}{2} \frac{W}{d} + W^2 > \frac{W}{d} \end{aligned}$$

$$\begin{aligned} W^2 - \frac{1}{2} \frac{W}{d} + \frac{1}{16d^2} &> 0 \\ \left(W - \frac{1}{4d}\right)^2 &> 0 \end{aligned}$$

which is true if $W \neq \frac{1}{4d}$ or $\omega - t - k - \gamma_S \neq \frac{1}{4p_A}$ but inequality (ii) implies this must hold.

Combining the inequality from the lemma and the owners' equal utility condition yields

$$\sqrt{(\omega - t - k - \gamma_S) / p_A} - (\omega - t - k - \gamma_S) < \frac{1}{2\sqrt{N_C}} + \gamma_S - \gamma_C + t$$

which can be rearranged as

$$\sqrt{(\omega - t - k - \gamma_S) / p_A} < \omega + \frac{1}{2\sqrt{N_C}} - k - \gamma_C.$$

This inequality states that nonowners get more utility from living in the city. QED.

When owners are at a corner solution they will be the only suburban residents. After including the city housing market clearing condition their equal utility condition becomes:

$$\begin{aligned} & \sqrt{(\omega + \sqrt{N_C} - t - \gamma_S) / p_A} + k \ln \left(2p_A \cdot \sqrt{-k + \sqrt{k^2 + (\omega + \sqrt{N_C} - t - \gamma_S) / p_A}} \right) \\ &= \omega + \sqrt{N_C} - k - \gamma_C + \frac{1}{2\sqrt{N_C}}. \end{aligned}$$

Unfortunately there is no closed-form solution for city population in terms of fundamental parameters.¹³ This intractability is due to the dependence of city population on parameters, such as ω , which drop out for interior solutions. That is, the separable, quasi-linear utility function simplifies the analysis enormously for interior solutions, but for a corner solution the parameters are tied together in complex ways. In particular the endowment of city land affects demand through income effects, which disappear when residents have interior solutions. Nonetheless, as the consumption constraint begins to bind owners in the suburb, so that marginal utility rises as housing and public good consumption falls, the city mayor can increase the rate of slack. That is, when the suburban mayor increases the rate of slack when its residents face corner solutions, the opportunity costs to residents become larger than in the case when residents can find interior solutions, which allows the city mayor to demand a higher rate of slack.

Cases in which some residents face corner solutions were they to live in the suburb could be analyzed either using calibration and numerical techniques or by adopting a model with outside ownership of all land. These approaches, however, are beyond the scope of this paper and are left for future research.

Appendix 2: Derivations

¹³The Maple kernel of Scientific Workplace was unable to solve this expression for city population. Attempts to do implicit differentiation, which could be used to analyze the mayors' reaction functions for corner solution cases, was also unsuccessful.

4.1 Corner Solutions: Omitted Calculations

$$s = \frac{\omega + p_C \cdot l_C(i) - t - 2kp_A\sqrt{s} - \gamma_S}{p_A}$$

$$s + 2k\sqrt{s} = \frac{\omega + p_C \cdot l_C(i) - t - \gamma_S}{p_A} \text{ Change variables to calculate: } x^2 + 2kx = \frac{\omega + p_C \cdot l_C(i) - t - \gamma_S}{p_A}$$

where $x = \sqrt{s}$

$$x^2 + 2kx - t = 0, \text{ Solution is: } \left\{ x = -t + \sqrt{(k^2 + t)} \right\}, \left\{ x = -k - \sqrt{(k^2 + t)} \right\}$$

$$s = -k + \sqrt{\left(k^2 + \frac{\omega + p_C \cdot l_C(i) - t - \gamma_S}{p_A} \right)} \text{ and } \tau_S^* = 2kp_A \sqrt{-k + \sqrt{\left(k^2 + \frac{\omega + p_C \cdot l_C(i) - t - \gamma_S}{p_A} \right)}}.$$

Only second solution is economically meaningful.

When owners are at a corner solution they will be the only suburban residents. After including the city housing market clearing condition their equal utility condition becomes:

$$\sqrt{\left(\omega + \sqrt{N_C} - t - \gamma_S \right) / p_A} + k \ln \left(2p_A \cdot \sqrt{-k + \sqrt{k^2 + \left(\omega + \sqrt{N_C} - t - \gamma_S \right) / p_A}} \right)$$

$$= \omega + \sqrt{N_C} + \frac{1}{2\sqrt{N_C}} - k - \gamma_C.$$

An analytic solution in terms of city population could not be obtained.

4.2 Derive population supply function from equal utility condition:

The equal utility condition implies

$$\frac{1}{2\sqrt{N_C}} = \frac{1}{4p_A} + (\gamma_C - \gamma_S) - t$$

so that

$$N_C(p_A, \gamma_C, \gamma_S, t, k) = \frac{1}{4 \cdot \left[\frac{1}{4p_A} + (\gamma_C - \gamma_S) - t \right]^2} = \frac{4p_A^2}{[1 + 4p_A \cdot [(\gamma_C - \gamma_S) - t]]^2} = \left(\frac{2p_A}{1 + 4p_A \cdot [\gamma_C - \gamma_S - t]} \right)^2$$

$$N_C(p_A, \gamma_C, \gamma_S, t, k, \varepsilon) = \left(\frac{2p_A}{1 + 4p_A \cdot [\gamma_C - \gamma_S - t]} \right)^2$$

4.3 Derivation of the City Mayor's Reaction Function

The derivative of city population w.r.t. city slack is

$$\frac{\partial N_C(\gamma_S, \gamma_C)}{\partial \gamma_C} = \frac{\partial \left[4 \left(\frac{p_A}{4p_A(\gamma_C - \gamma_S - t) + 1} \right)^2 \right]}{\partial \gamma_C} = 32 \frac{p_A^3}{(4p_A t - 4p_A \gamma_C + 4p_A \gamma_S - 1)^3}$$

and the elasticity of population w.r.t. city slack is

$$\frac{\partial N_C(\gamma_S, \gamma_C)}{\partial \gamma_C} \cdot \frac{\gamma_1}{N_C(\gamma, \gamma_1)} = 32 \frac{p_A^3}{(4p_A t - 4p_A \gamma_C + 4p_A \gamma_S - 1)^3} \cdot \frac{\gamma_1}{\left[4 \left(\frac{p_A}{1 + 4p_A(\gamma_C - \gamma_S - t)} \right)^2 \right]} = \frac{8\gamma_1 p_A}{\frac{(4p_A t - 4p_A \gamma_C + 4p_A \gamma_S - 1)^3}{(1 + 4p_A(\gamma_C - \gamma_S - t))^2}}$$

$$= \frac{-8p_A \gamma_1}{4p_A(t - \gamma_1 + \gamma_2) - 1}$$

The city mayor's reaction function is then characterized by the condition:

$$\begin{aligned}\frac{\partial N_C(\gamma_C, \gamma_S)}{\partial \gamma_C} \frac{\gamma_C}{N_C} &= \frac{8p_A \cdot \gamma_C}{4p_A(-t + \gamma_C - \gamma_S) + 1} = -1 \\ 8\gamma_C p_A &= [4p_A(-t + \gamma_C - \gamma_S) + 1] \\ \gamma_C(\gamma_S; t, p_A) &= (1/4p_A - t - \gamma_S, 0)\end{aligned}$$

4.4 Derivation of the Suburban Mayor's Reaction Function

The derivative of suburban population w.r.t. suburban slack is

$$\begin{aligned}\frac{d}{d\gamma_S} \left[\gamma_S \cdot \left(1 - \left[\frac{2p_A}{1+4p_A(\gamma_C - \gamma_S - t)} \right]^2 \right) \right] \\ \gamma_S(\gamma_C; t, p_A) = \frac{-p_A t + p_A \gamma_C - \frac{1}{4}\gamma_C + \frac{1}{4} + \frac{1}{4} \sqrt{(8p_A t \gamma_C - 8p_A \gamma_C^2 + \gamma_C^2 - 2\gamma_C + 4p_A^2)}}{p_A}\end{aligned}$$



let $w = (\gamma_C - t)$

$$\begin{aligned}\frac{d}{d\gamma_S} \left[\gamma_S \cdot \left(1 - \left[\frac{2p_A}{1+4p_A(w - \gamma_S)} \right]^2 \right) \right] &= \left(1 - \left[\frac{2p_A}{1+4p_A(w - \gamma_S)} \right]^2 \right) + \gamma_S \cdot \frac{d}{d\gamma_S} \left[\left(1 - \left[\frac{2p_A}{1+4p_A(w - \gamma_S)} \right]^2 \right) \right] \\ &= 1 - \left[\frac{2p_A}{1+4p_A(w - \gamma_S)} \right]^2 - 4\gamma_S \cdot \left[\frac{2p_A}{1+4p_A(w - \gamma_S)} \right]^3 \\ &= \frac{-1 - 12p_A w + 12p_A \gamma_S - 48p_A^2 w^2 + 96p_A^2 w \gamma_S - 48p_A^2 \gamma_S^2 - 64p_A^3 w^3 + 192p_A^3 w^2 \gamma_S - 192p_A^3 w \gamma_S^2 + 64p_A^3 \gamma_S^3 + 4p_A^2 + 16p_A^3 w + 16\gamma_S p_A^3}{(-1 - 4p_A w + 4p_A \gamma_S)^3} = \\ &0,\end{aligned}$$

Solution is: $\left\{ \gamma_S = \frac{1}{p_A} \left(\frac{1}{12} \sqrt[3]{X} - \frac{p_A^2}{\sqrt[3]{X}} + \frac{1}{4} + p_A w \right) \right\},$
 $\left\{ \gamma_S = \frac{1}{p_A} \left(-\frac{1}{24} \sqrt[3]{X} + \frac{1}{2} \frac{p_A^2}{\sqrt[3]{X}} + \frac{1}{4} + p_A w + \frac{1}{2} i \sqrt{3} \left(\frac{1}{12} \sqrt[3]{X} + \frac{p_A^2}{\sqrt[3]{X}} \right) \right) \right\},$
 $\left\{ \gamma_S = \frac{1}{p_A} \left(-\frac{1}{24} \sqrt[3]{X} + \frac{1}{2} \frac{p_A^2}{\sqrt[3]{X}} + \frac{1}{4} + p_A w - \frac{1}{2} i \sqrt{3} \left(\frac{1}{12} \sqrt[3]{X} + \frac{p_A^2}{\sqrt[3]{X}} \right) \right) \right\}$
where $X \equiv \left(-108p_A^2 - 432p_A^3 w + 12\sqrt{(12p_A^6 + 81p_A^4 + 648p_A^5 w + 1296p_A^6 w^2)} \right)$
choose root with all real parts:
 $\left\{ \gamma_S = \frac{1}{p_A} \left(\frac{1}{12} \sqrt[3]{X} - \frac{p_A^2}{\sqrt[3]{X}} + \frac{1}{4} + p_A w \right) \right\},$
 $\gamma_S () = \max \left(0, \frac{1}{p_A} \left(\frac{1}{12} \sqrt[3]{X} - \frac{p_A^2}{\sqrt[3]{X}} + \frac{1}{4} + p_A w \right) \right)$

4.5 Nash Equilibria

Find analytic expression for Nash equilibrium values:

$$\gamma_S(\gamma_C) = \frac{1}{12p_A} \sqrt[3]{X2} - \frac{p_A}{\sqrt[3]{X}} + \frac{1}{4p_A} + (\gamma_C - t)$$

where

$$X2 \equiv \left(-108p_A^2 - 432p_A^3 (\gamma_C - t) + 12\sqrt{(12p_A^6 + 81p_A^4 + 648p_A^5 (\gamma_C - t) + 1296p_A^6 (\gamma_C - t)^2)} \right)$$

$$\gamma_C(\gamma_S; t, p_A) = \left(\frac{1}{4p_A} - t - \gamma_S, 0 \right) \text{ invert city mayor's reaction function } \left\{ \gamma_S = \frac{1}{4p_A} - t - \gamma_C \right\}$$

Solve for city mayor's Nash equilibrium value:

$$\frac{1}{4p_A} - t - \gamma_C = \frac{1}{12p_A} \sqrt[3]{X2} - \frac{p_A}{\sqrt[3]{X}} + \frac{1}{4p_A} + (\gamma_C - t),$$

An analytic solution not found.

4.6 Calculations

City rental rate as a function of fundamentals: illustrative case when $p_A = 1/3$

$$p_C^*(p_A, t) = \left[\frac{1}{\frac{1}{p_A} - \left(\frac{4}{7}t + \frac{12}{7} + \frac{2}{7}\sqrt{(4t^2 - 4t + 29)} \right)} \right]_{p_A=1/3}$$

Not defined where $\frac{1}{p_A} = \frac{4}{7} \left(t + 3 + \frac{1}{2}\sqrt{(4t^2 - 4t + 29)} \right)$. Note that all live in the city if

$$\gamma_S - \gamma_C + t + k \cdot \ln(1 + \varepsilon) \geq \frac{1}{4p_A} - \frac{1}{2}.$$

City rental rate as a function of agric. rental rate, three illustrative values.

$$p_C^*(p_A, t) = \left[\frac{1}{\frac{1}{p_A} - \frac{4}{7} \left(t + 3 + \frac{1}{2}\sqrt{(4t^2 - 4t + 29)} \right)} \right]_{t=1/4} = \frac{1}{-\frac{13}{7} - \frac{1}{14}\sqrt{113}\sqrt{4} + \frac{1}{p_A}} = \frac{1}{-\frac{13}{7} - \frac{2}{14}\sqrt{113} + \frac{1}{p_A}}$$

$$p_C^*(p_A, t) = \left[\frac{1}{\frac{1}{p_A} - \frac{4}{7} \left(t + 3 + \frac{1}{2} \sqrt{(4t^2 - 4t + 29)} \right)} \right]_{t=1/3} = \frac{1}{\frac{1}{p_A} - \frac{40}{21} - \frac{2}{63} \sqrt{253} \sqrt{9}} = \frac{1}{\frac{1}{p_A} - \frac{40}{21} - \frac{6}{63} \sqrt{253}}$$

$$p_C^*(p_A, t) = \left[\frac{1}{\frac{1}{p_A} - \frac{4}{7} \left(t + 3 + \frac{1}{2} \sqrt{(4t^2 - 4t + 29)} \right)} \right]_{t=1/2} = \frac{1}{\frac{1}{p_A} - 2 - \frac{4}{7} \sqrt{7}}$$

vary agr costs, look at effect on transportation

$$p_C^*(p_A, t) = \left[\frac{1}{\frac{1}{p_A} - \frac{4}{7} \left(t + 3 + \frac{1}{2} \sqrt{(4t^2 - 4t + 29)} \right)} \right]_{p_A=1/6} = \frac{1}{\frac{30}{7} - \frac{4}{7} t - \frac{2}{7} \sqrt{(4t^2 - 4t + 29)}}$$

$$p_C^*(p_A, t) = \left[\frac{1}{\frac{1}{p_A} - \frac{4}{7} \left(t + 3 + \frac{1}{2} \sqrt{(4t^2 - 4t + 29)} \right)} \right]_{p_A=1/5} = \frac{1}{\frac{23}{7} - \frac{4}{7} t - \frac{2}{7} \sqrt{(4t^2 - 4t + 29)}}$$

$$p_C^*(p_A, t) = \left[\frac{1}{\frac{1}{p_A} - \frac{4}{7} \left(t + 3 + \frac{1}{2} \sqrt{(4t^2 - 4t + 29)} \right)} \right]_{p_A=1/4} = \frac{1}{\frac{16}{7} - \frac{4}{7} t - \frac{2}{7} \sqrt{(4t^2 - 4t + 29)}}$$

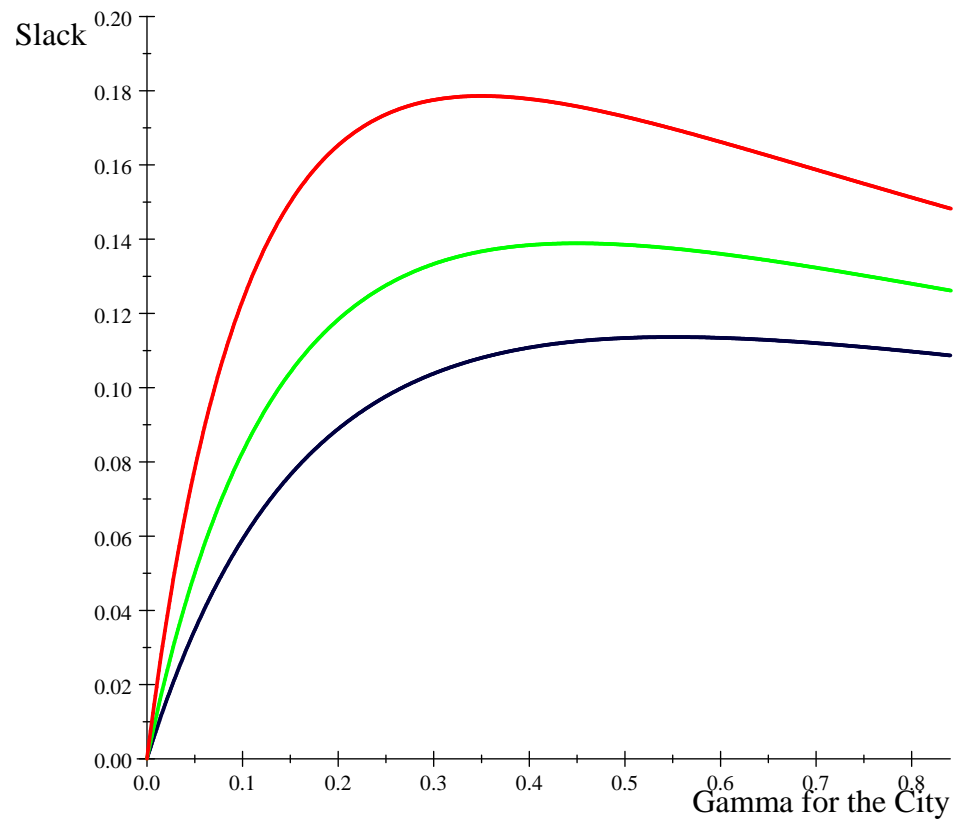


Figure 1: A Family of Total Slack Functions for the City Mayor

Legend: From the lowest to highest lines: ; $t = 0.5$; $t = 0.6$; $t = 0.7$. For all lines shown here $p_A = 0.2$; $\gamma_S = 0.2$; $\varepsilon = 0$

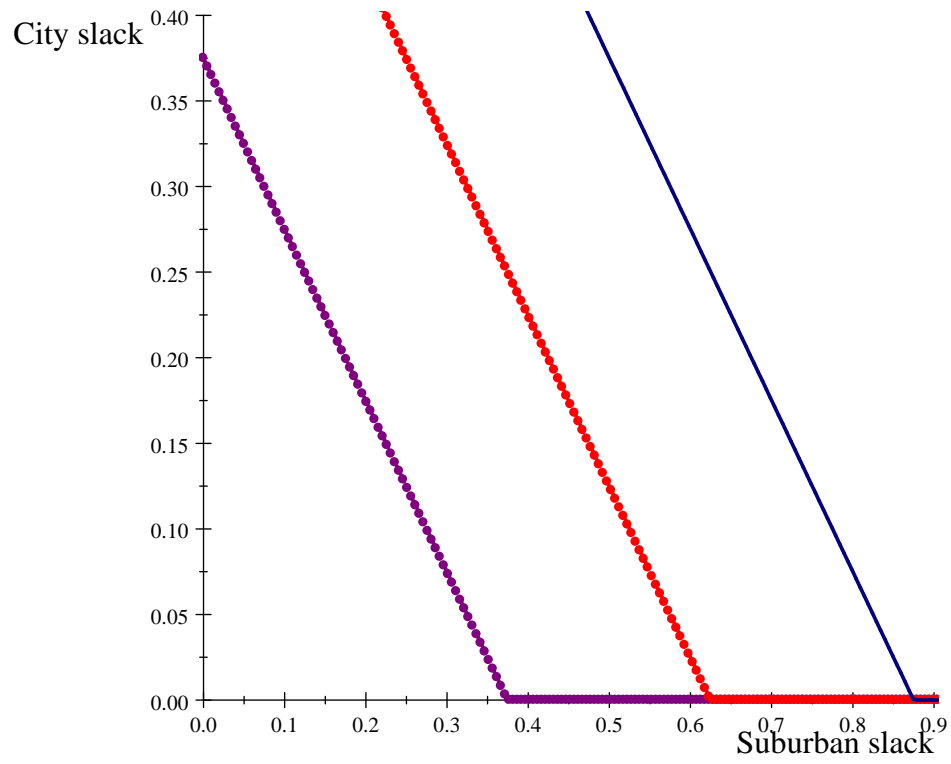


Figure 2: City Mayor's Reaction Function

Legend: $p_A = 1/2$ for dashed line; $p_A = 1/3$ for dot-dashed line; and $p_A = 1/4$ for the solid line; $t = 1/8$ for all lines.

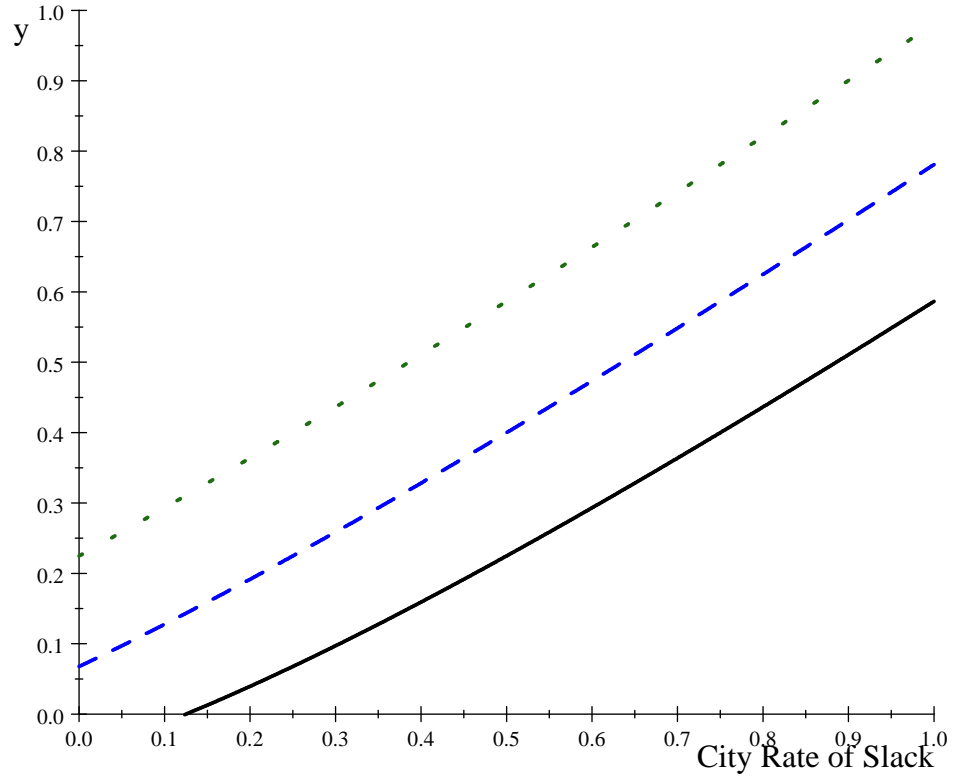


Figure 3: Suburban Mayor's Reaction Function

Legend: $p_A = 1/2$ for solid line; $p_A = 1/3$ for dot-dashed line; and $p_A = 1/4$ for the dotted line; $t = 1/8$ for all lines. Vertical axis represents the suburb's rate of slack.

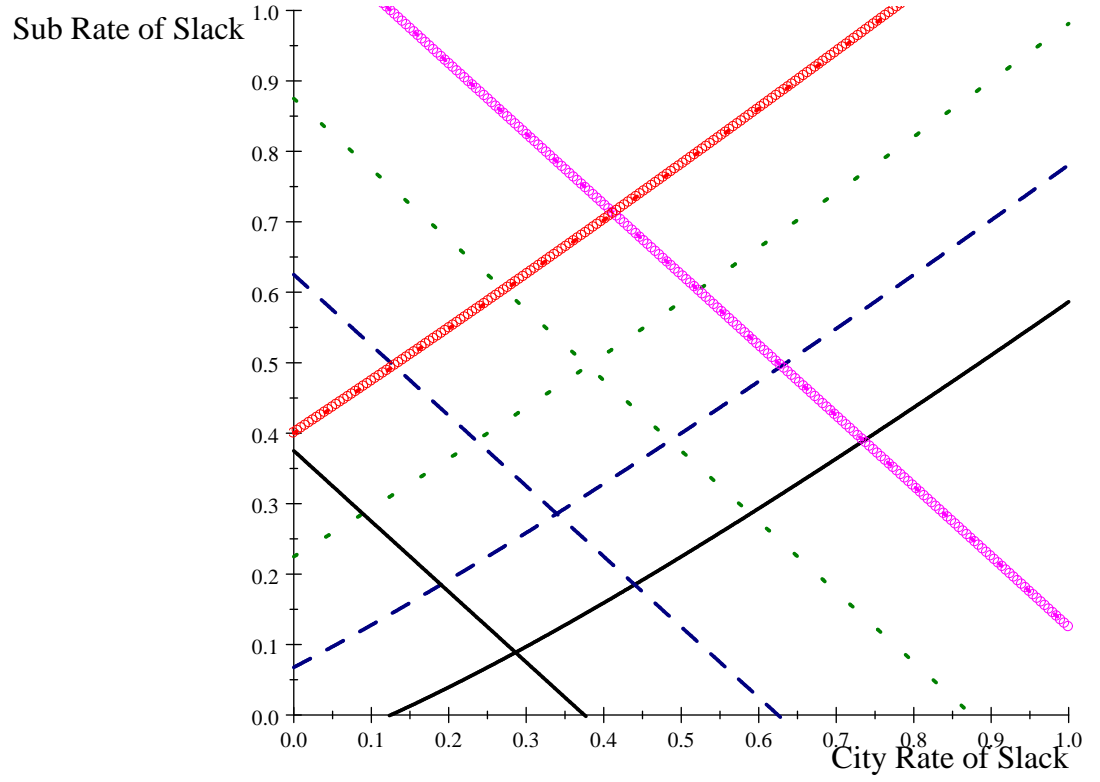


Figure 4: Nash Equilibria for Various Levels of Agricultural Rent

Legend: From bottom to top: $p_A = 1/2$ for solid lines; $p_A = 1/3$ for dashed lines; and $p_A = 1/4$ for the dotted lines; $p_A = 1/5$ for the dot-dashed lines; $t = 1/8$ for all lines. Vertical axis represents the suburb's rate of slack.

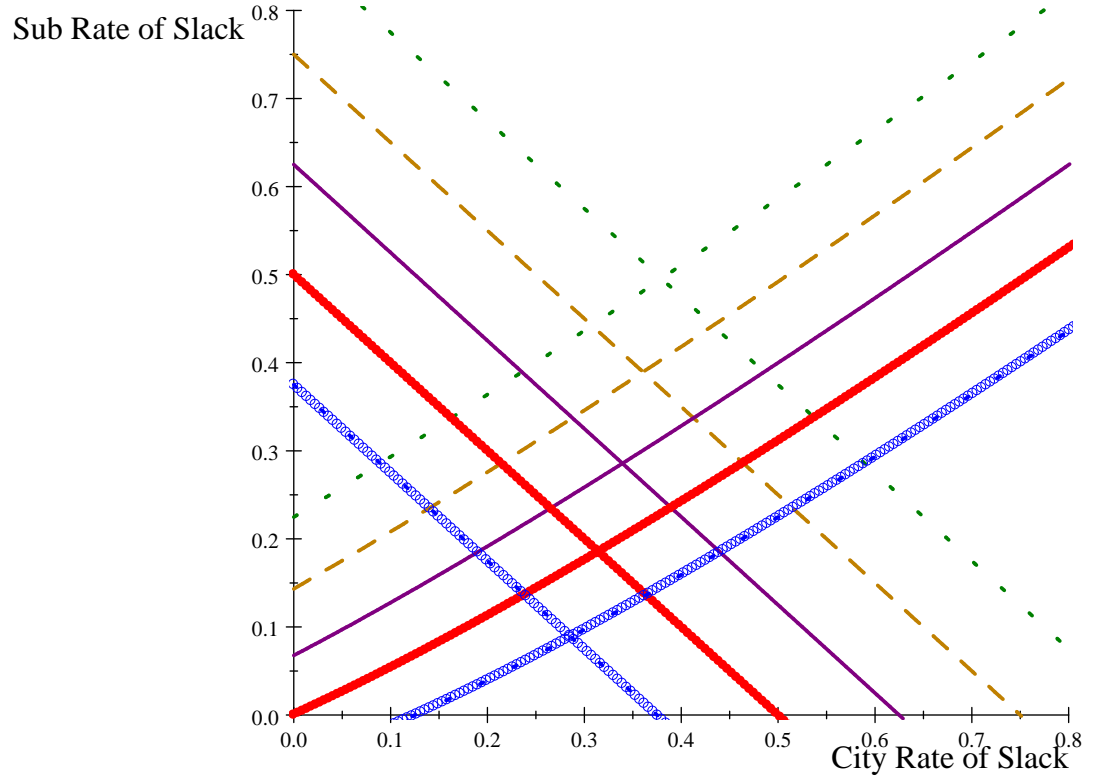


Figure 5: Nash Equilibria for Various Transportation Costs

Legend: From top to bottom: $t = 1/8$ for dotted lines; $t = 1/4$ for dashed lines; $t = 3/8$ for solid lines; $t = 1/2$ for dot-dashed lines; $t = 5/8$ for the lowest set of lines; $p_A = 1/4$ for all cases.

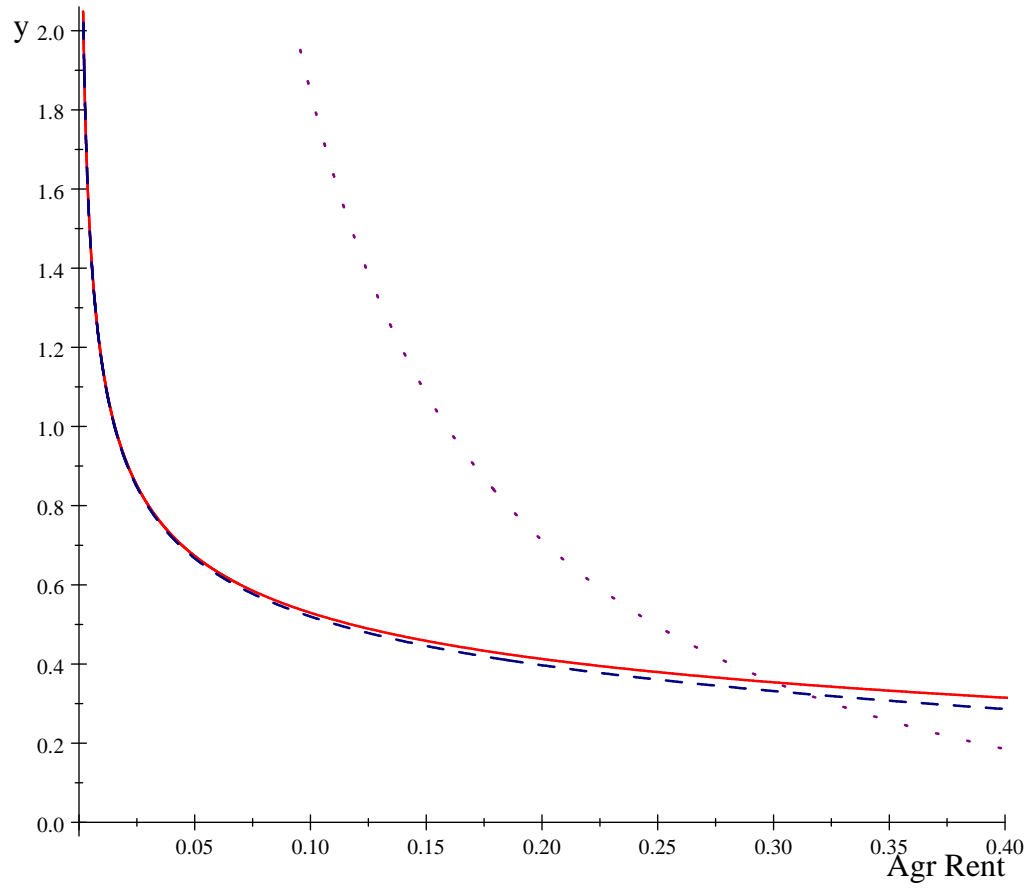


Figure 6: Equilibrium City Rate of Slack as Function of Agr. Rent

Legend: Solid line shows city rate of slack in Nash equilibrium when $t = 1/8$; dotted line shows suburban rate of slack when $t = 1/8$; dashed line shows $t = 1/4$. Vertical axis represents the city rate of slack.

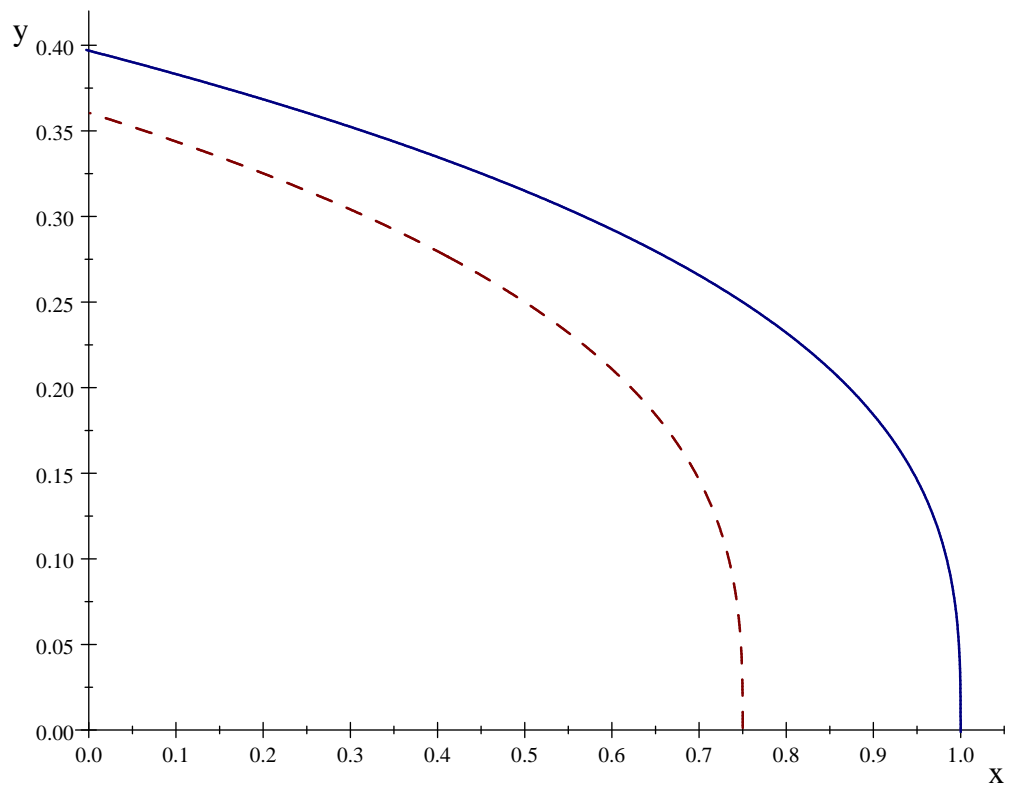


Figure 7: Equilibrium City Rate of Slack as Function of Transport Costs

Legend: Solid line shows $p_A = 1/4$; dashed line shows $p_A = 1/3$; vertical axis shows rate of city slack.

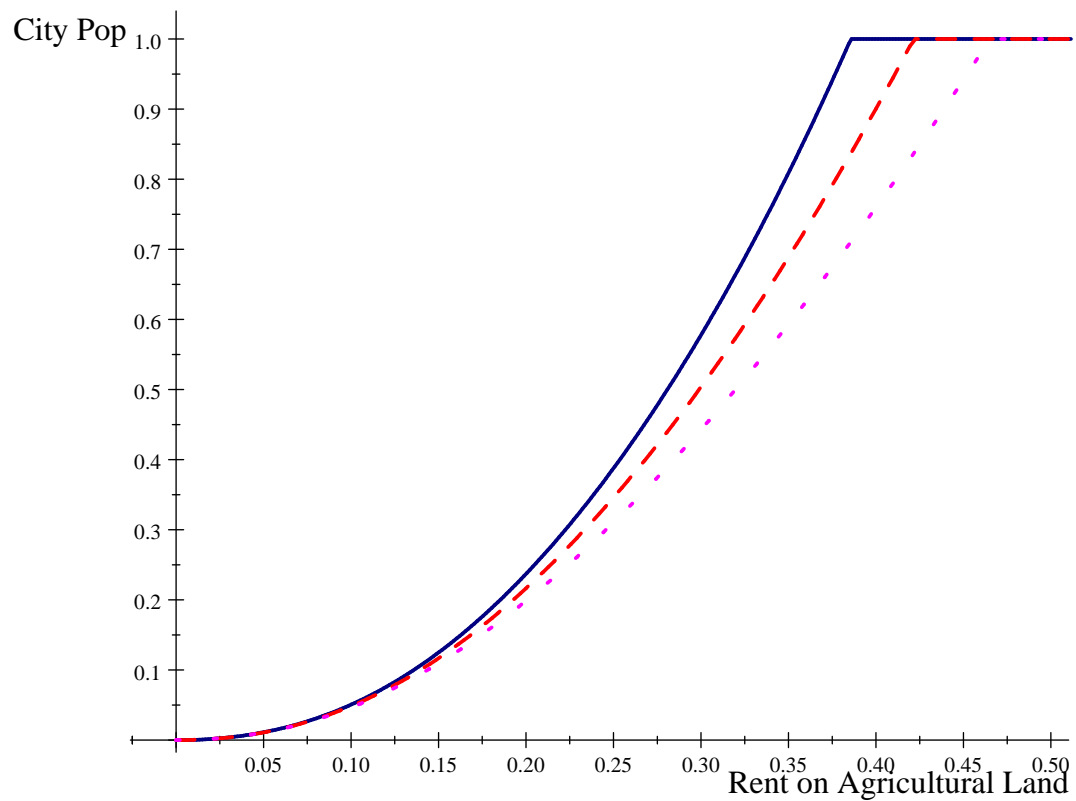


Figure 8: City Population as a Function of Agricultural Rent

Legend: Solid line shows the population function for $t = \frac{3}{10}$; dashed line shows $t = 1/4$; dotted line shows $t = 1/5$.

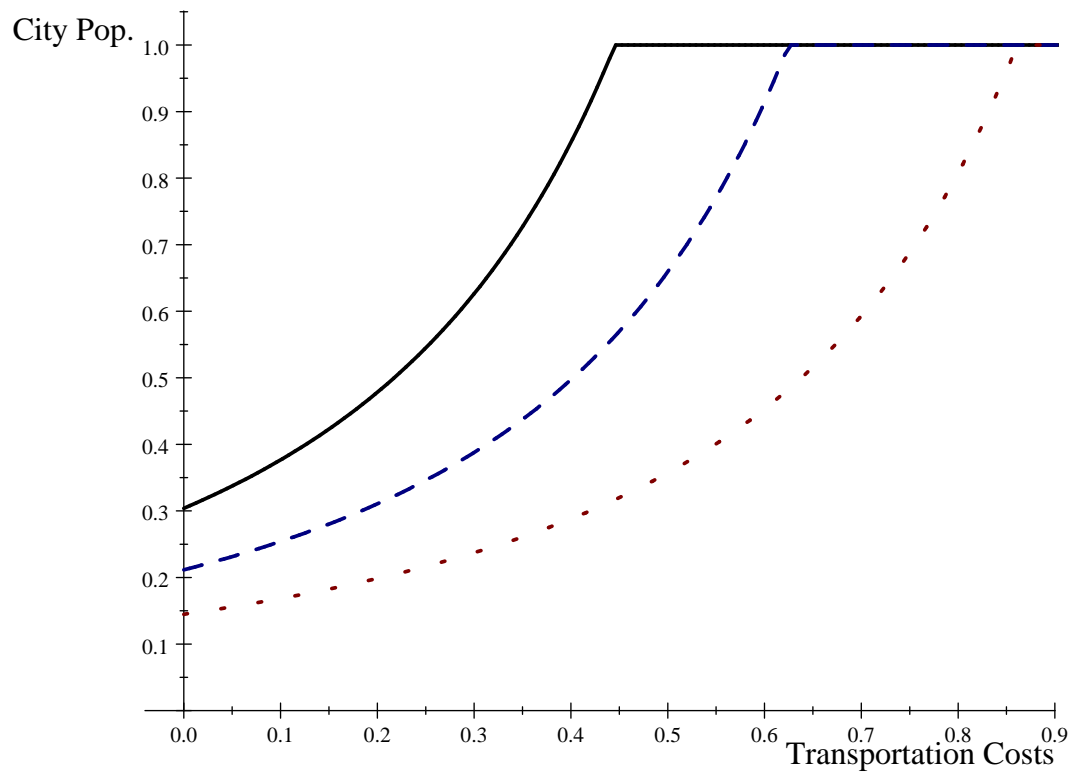


Figure 9: City Population as Function of Transport Cost

Legend: Solid line shows $p_A = 1/3$; dashed line shows $p_A = 1/4$; dotted line shows $p_A = 1/5$.

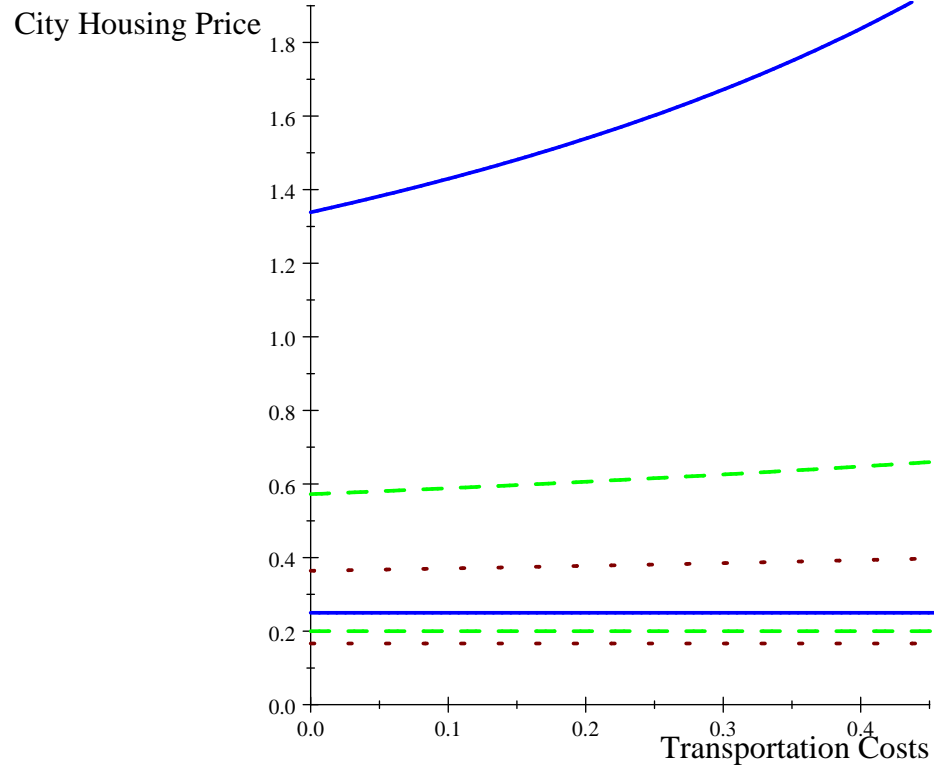


Figure 10: Equilibrium Price of City Housing as a Function of Transportation Costs

Legend: The solid line drawn for city rental rate when $p_A = 1/4$; dashed line when $p_A = 1/5$; dotted line when $p_A = 1/6$.

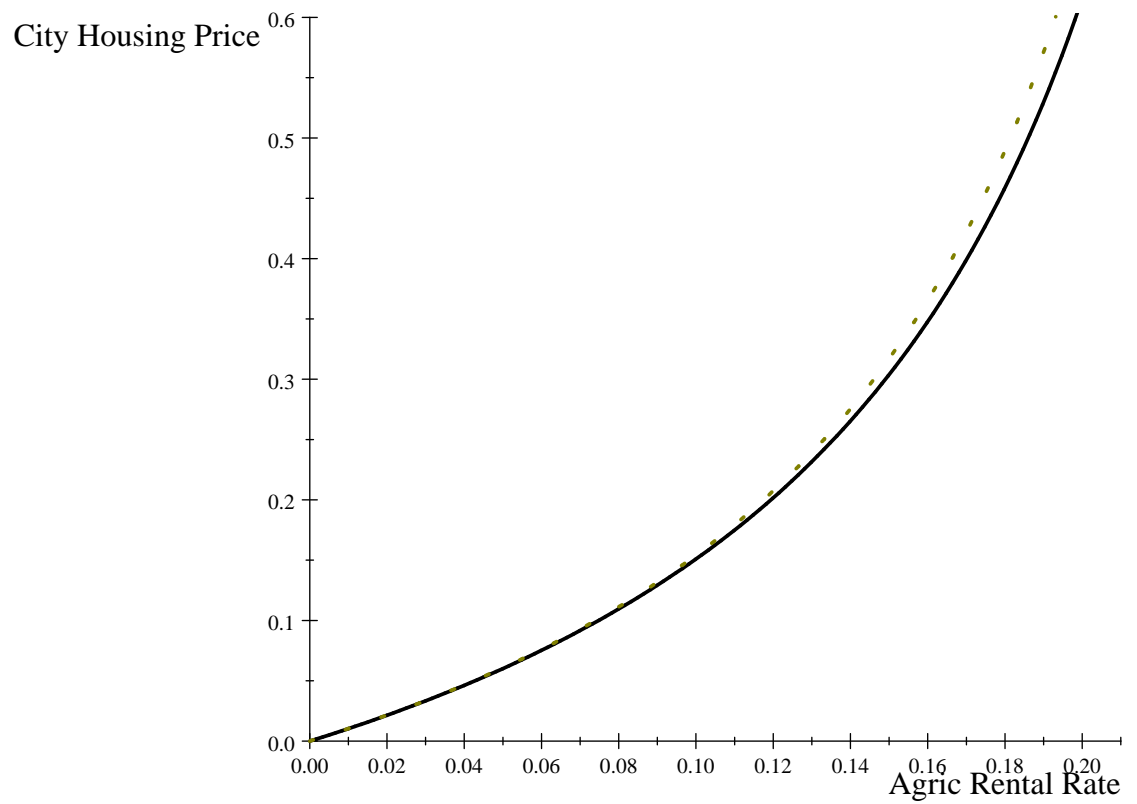


Figure 11: Equilibrium Price of City Housing as a Function of Agricultural Rental Rate

Legend: The solid line shows $t = \frac{1}{4}$; dotted line shows $t = 1/2$.
Appendix

