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Consistent Estimation of Portfolio Variance and the Benefits of Diversification. A Cautionary Note

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October 13, 2005

Abstract

Unbiased and consistent estimation of portfolio volatility requires data to satisfy some conditions related with stationarity of the time series, in order to make tractable the behavior of sample moments of the assets return distributions. In such a context, using returns along with multivariate GARCH models has proven to be sterile specially when transactions are infrequent. This, implies that risk assessment based on standard approaches might be misled when data problems are not considered. An obvious result is that cross relations between assets remain unknown until a consistent estimation is achieved, which has important consequences for several financial applications.

In this paper we briefly show how to overcome those problems that occur in the presence of microstructure effects through the use of a range-based volatility estimator. Conditional variances and covariances are obtained using the LCARR(p,q) model of Borquez (2005) which allows for a simple multivariate setting (MLCARR) when is estimated in conjunction with the constant conditional correlation assumption of Bollerslev (1990).

Empirical evidence is then focused on comparing estimations of the correlation matrix using the MLCARR model and a number of multivariate GARCH models, over a portfolio including a sample of nine stocks traded in the chilean market. Those have been choosen to reflect variability in the supply with high, medium and low capitalized assets considered.

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Results are straightforward. Data problems cause correlations to be severe downward biased meaning that benefits of diversification could be overestimated by returns-based volatility estimations. Moreover, the bias is stable across time for all GARCH models indicating that problems arise from complex autoregressive non-stationarity and therefore they are more likely to be solved by choosing an adequate volatility proxy.

From a regulatory perspective, this implies that risk related policies are being designed assuming a low risk environment, which could limit their response capacity to high risk events, precisely when they are more important. In addition, overestimating diversification would lead to rieskier unbalanced portfolios with severe consequences for risk management.

1 Introduction

Volatility is a key ingredient for many applied issues in financial and financial engineering, such as asset pricing, asset allocation and risk management. Estimates of return volatility are used to assess the risk of many financial products and accurate measures and reliable forecasts of volatility are crucial for derivative pricing techniques, as well as trading and hedging strategies that arise in portfolio allocation problems.

Since Fama (1965) and Mandelbrot (1963) we know that financial returns are influenced by time dependent information flows which results in pronounced temporal volatility clustering. In such a context, unbiased and consistent estimation of volatility requires data to satisfy some conditions related with stationarity of the time series, in order to make tractable the behavior of sample moments of the assets return distributions.

To explain this, let us consider the following random-walk model for the observed log price (x) of a security

$$\varepsilon_t = x_t - x_{t-1} \; ; \; \varepsilon_t | \Omega_{t-1} \sim N(0, \sigma^2)$$

The capacity of squared returns (ε_t^2) as a proxy of the second moment of the underlying process critically rests on exploiting the assumption that all the relevant information Ω is captured by the conditional variance at time $t-1$. However, if volatility is not constant over time, then the representation above results inappropriate and some additional structure is needed to adequately characterize the conditional expectation. This is more likely to occur in the presence of microstructure effects (i.e. infrequent trading, bid-ask spreads, low volume, etc.) causing data to be serially dependent.

Even if observed returns are stationary, volatility estimation and forecasting could be seriously affected by the sample process. For example, assume that observed returns exhibit the following AR(1) process $\varepsilon_t = \eta\varepsilon_{t-1} + \xi_t$ where ξ_t is a white noise with variance equal to σ^2. The serie is covariance-stationary if $|\eta| < 1$ but the conditional variance is now a function of its past values, therefore we have $E\left[\varepsilon_t^2\right] = \frac{\sigma^2}{1-\eta^2} > \sigma^2$. Moreover, we can represent this unconditional variance as a MA(∞) where $E\left[\varepsilon_t^2\right] = E(\xi_t + \psi_1\xi_{t-1} + ... + \psi_T\xi_{t-T})^2 = E(1 + \psi_1^2 + ... + \psi_T^2)\sigma^2$ given that $\sum_{j=1}^{\infty}\psi_j^2 < \infty$. It is easy to show that the limit probability of $E\left[\varepsilon_t^2\right]$ is greater than σ^2, for which volatility estimations and forecasting are not only unbiased but also inconsistent in the presence of microstructure effects.

Consequently, volatility estimation procedures varies a great deal depending on how much information we have at each time. For instance, several representations of GARCH models are available allowing to capture specific information of the time series, such as asymmetries, long memory or excess of kurtosis in volatility to name a few. Nevertheless, data problems encompasses biased and inconsistent estimation when are based on returns (absolute or squared) as they are greatly affected by microstructure problems.

An alternative approach is then to focus on the range, the difference of the logarithmic high and low prices. Parkinson (1980) proved that the range is related with the distribution function of the price of an asset that follows a random walk. He also, demonstrated that the return variance can be directly estimated from extreme values in a superior manner (more efficient) than using open and closing prices (as in the case of squared returns). Brand and Diebold (2002) refers to range as a highly efficient volatility estimator quite robust to microstructure effects.

Chow (2005) states that not considering the conditional behavior of the range would explain the empirical failure that has characterized this proxy in the past. In that sense, this author developed a conditional model for the range, named Conditional Autoregressive Range (CARR) model, using a GARCH representation to allow for conditional time-dependence. Alizadeh et al. (2002) used the logarithm of the range as a proxy for estimating stochastic volatility models. Finally, Borquez (2005) extended the CARR model to consider the log-range in a GARCH setting.

Range-based volatility measures have been characterized by a common problem, its univariate nature. To solve this, Brandt and Diebold (2002) proposed a no-arbitrage strategy to estimate covariances in the context of high frequency data, that is used by Brunetti and Lildholdt (2002) for their multivariate extension of the Parkinson's volatility estimator, the co-range. There is, however, a serious limitation on accessing intraday data in less developed financial markets that erodes the possibility of using those previous approaches. Nevertheless, Bórquez (2005) showed that differences of the log-range estimator respect to its mean are sufficient to capture co-movements between assets because that proxy is approximately distributed as normal with zero mean. Using this property, the LCARR model can be easily formulated to the multivariate case incorporating the constant conditional correlation assumption of Bollerslev (1990).

Our objective is then to study the effect of a consistent estimation of the portfolio variance in the context of infrequent trading, respect to more traditional approaches such as GARCH models and Riskmetrics which use returns as a volatility proxy. The rest of the paper is organized as follows. Section 2 reviews several conditional volatility models which are estimated in section 3. Section 4 presents our empirical results. In Section 5 we conclude, briefly discussing the implications for financial applications focusing on portfolio management and risk assessment.

2 Estimating Volatility and Consistency

A simple way to incorporate actual data into the estimation of time-varying volatility is by using rolling sample windows (Andreou and Ghysels, 2002; Andersen et. al., 2005). In this case, the variance is based on the p most recent observations is $\widehat{\sigma}_t^2 = \frac{1}{p}\sum_{i=0}^{p-1}(\varepsilon_i - \overline{\varepsilon})^2$. If $\widehat{\sigma}_t^2$ is an estimate of the current variance of ε_t, the value of p directly determines the variance-bias of the estimator, with larger values of p reducing the variance but increasing the bias. Notice that the bias of the estimator may be reduced by assigning more weights to the most recent observations. In particular, let us consider the following variance estimator commonly used in financial institutions, the exponentially moving average filter:

$$\widehat{\sigma}_t^2 = \gamma(\varepsilon_i - \overline{\varepsilon})^2 + (1-\gamma)\widehat{\sigma}_{t-1}^2 = \gamma\sum_{i=0}^{\infty}(1-\gamma)^{i-1}\varepsilon_{t-i}^2,$$

where $\overline{\varepsilon}$ is the mean of returns and γ is a parameter whose value is determined by experience. The sum will have to be truncated at $i = t-1$, which is frequently done by equating the pre-sample values to zero and adjusting the finite sum by the corresponding multiplicative factor $1/\left[1-(1-\gamma)^t\right]$. This approach is exemplified by Riskmetrics which rely on a value of $\gamma = 0.06$ and $\overline{\varepsilon} = 0$ in their construction of daily volatility measures.

These data-driven filters are often used in place of more formal model building procedures in the construction of $h-$period-ahead volatility forecasts, by simply equating the future volatility of interest with the current filtered estimate $Var(\varepsilon_{t+h}/\Omega_{t-1}) = \sigma_{t+h|t}^2 = \widehat{\sigma}_t^2$. However, it is difficult to contemplate optimal volatility forecasts without the notion of a model or data generating process, and they are incapable to optimally describe the conditional characteristics of volatility as they can not be considered as a tool for short run forecasts[1]. In fact, Andersen et. al. (2005) shown that the theoretical properties of these filters as a consistent method to extract volatilities are valid only for very low frequency data, where the estimation based on weighted moving average may be optimal.

Traditional econometric models consider a constant forecast variance, and it was not until 1982 when Robert Engle introduced his $ARCH$ model (Engle, 1982), that modelled the time variation of second - and higher - order moments really took off. One of the key ideas behind the whole family of $ARCH$ models is that models that make use of recent available information will be able to forecast better than other models that do not take into account this information through explicitly modeling second order dependencies. Conventional econometric models do not allow for a conditional variance whose values depend on past information, so volatility clustering is not a phenomenon that can be understood with the aid of these traditional models.

The main characteristics of $ARCH$ processes are that they assume that the innovations are serially uncorrelated, have a mean zero, a constant unconditional variance, and most importantly, they have non-constant variances conditional on the past.

In the case of a simple first-order autoregression

$$y_t = \alpha y_{t-1} + \varepsilon_t,$$

where ε_t is a white noise with variance $E(\varepsilon_t^2) = \sigma^2$, the unconditional mean of y_t is zero, while its conditional mean is αy_{t-1}. It has unconditional variance $Var(y_t) = \sigma^2$, and its conditional variance is $Var(y_t | y_{t-1}) = \frac{\sigma^2}{1-\alpha^2}$, so the variance of this model is in both cases constant. In order to model a variance that is not constant over time, the approach given by Engle was to propose the following model

$$\begin{aligned} y_t &= \varepsilon_t \sigma_t \\ \sigma_t^2 &= \alpha_0 + \alpha_1 y_{t-1}^2, \end{aligned} \tag{1}$$

where $\varepsilon_t \sim iid(0,1)$ (white noise with unit variance), and σ_t is a positive time-varying function of Ω_{t-1}, the information set available at time $t-1$.

The variance defined in (1) can be generalized in order to include information that goes further back than only one period; that is

$$\sigma_t^2 = h(1, y_{t-1}, y_{t-2}, \ldots, y_{t-q}, \boldsymbol{\alpha}),$$

where $\boldsymbol{\alpha} = (\boldsymbol{\alpha}_0, \alpha_1, \ldots, \alpha_q)'$, which defines the $ARCH(q)$ model, where $\boldsymbol{\alpha}$ is a vector of unknown parameters.

[1] RiskMetrics is not really a data generating process and consequently it has associated no condtional expectation or forecast.

To obtain the $ARCH$ regression model, the mean of y_t is assumed to be a linear combination of lagged variables included in the information set at time $t-1$, that is $E(y_t) = x_t'\beta$ where β is a vector of unknown parameters. Formally, the ARCH regression model can be written as

$$
\begin{aligned}
y_t \,/\Omega_{t-1} &\sim N(x_t'\beta, \sigma^2) \\
y_t &= x_t'\beta + \varepsilon_t \\
\sigma_t^2 = h(\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots, \varepsilon_{t-q}, \boldsymbol{\alpha}) &= \alpha_0 + \textstyle\sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2.
\end{aligned} \tag{2}
$$

If $\alpha_i = 0$, $i = 1, 2, ..., q$, then the $ARCH(1)$ process become Gaussian white noise.

The $ARCH(q)$ model was a major breakthrough in econometric modelling, however, in several applications a problem arose because a long lag length q was needed in order to satisfactorily explain certain data. This motivated Bollerslev (1986) to introduce a parsimony model with different -more flexible- lag structure. The *generalized* $ARCH$ or $GARCH(p, q)$ model is

$$
\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \phi_j \sigma_{t-j}^2. \tag{3}
$$

A direct comparison between equations (3) and (2) reveals that the $GARCH$ is simply an extension of the $ARCH$ model, which allows for the variance to be dependent on past values itself. If the $GARCH$ process defined in (3) is represented as an infinite-order AR; i.e., $\sigma_t^2 = \boldsymbol{\psi}(L)\varepsilon_t^2 = (1 - \boldsymbol{\phi}(L))^{-1}\boldsymbol{\alpha}(L)\varepsilon_t^2$, then in order for the process to be well defined all the parameters in the AR representation must be nonnegative. Specifically, for the case of a $GARCH(1, 1)$, the condition is $\alpha_1 \geq 0$ and $\phi_1 \geq 0$.

Regarding stationarity, we can say that the $GARCH(1, 1)$ process is covariance stationary if and only if $\alpha_1 + \phi_1 < 1$, a situation that is exactly the same as having an $ARCH$ process of infinite-order where the parameters decline geometrically.

Another important feature of the $GARCH(p, q)$ model is that it can be regarded as an $ARMA(r, p)$, where r is the $\max(p, q)$. This result is important not only because it allows to apply the analysis of $ARMA$ processes to the $GARCH$ model, but also because it is known that such an $ARMA$ process has a unit root when

$$
\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \phi_j = 1. \tag{4}
$$

A model that satisfies the condition given in (4) is known as an *integrated* $GARCH$ process or $IGARCH$ (Engle and Bollerslev, 1986). Although a process y_t that follows an

$IGARCH$ process is not covariance stationary, and its unconditional variance is infinite, an $IGARCH$ process is still important since the unconditional density of y_t is the same for all t, and thus the process y_t can come from a strictly stationary process.

The key concept behind the above heteroskedastic models was that σ_t should be a positive time-varying function of the information set at time $t-1$ (i.e., $\sigma_t(\Omega_{t-1})$). However, in both the ARCH and GARCH models, the mean of the process is not affected by the conditional variance, a fact that is the main idea in the $ARCH - in - Mean$ $(ARCH - M)$ model.

Conditional variances in both $ARCH$ and $GARCH$ are expressed as linear function of innovations, but in markets where prices are considered to be Martingales, price changes should also be considered as innovations and the means and variances of returns that are observed should be related, thus moving in the same direction although not in the same proportion (Engle et. al., 1987). In the $ARCH-M$ model the disturbances are heteroskedastic but the model includes additional information since the standard deviation seen at each time t, will affect the mean of that same observation. Formally, the model is

$$\begin{aligned} y_t \,/x_t, \pi_t &\sim N(\beta' x_t + \delta\sqrt{h_t}, h_t) \\ h_t &= \boldsymbol{\alpha}'\mathbf{W}_{\eta_t} + \boldsymbol{\gamma}' Z_t \end{aligned} \tag{5}$$

where weakly exogenous and lagged variables are represented by the $k \times 1$ vector x_t and the $m \times 1$ vector Z_t. In the last vector the constant variance component of $\sqrt{h_t}$ is included. Disturbances ε_t given by $y_t - \beta' x_t - \delta\sqrt{h_t}$ are presented in the form of the $p \times 1$ vector $\eta_t' = (\varepsilon_{t-1}^2, \varepsilon_{t-2}^2, \ldots, \varepsilon_{t-q}^2)$. The $q \times p$ matrix $\mathbf{W}$ is, in the completely unrestricted case, the identity matrix. It is a matrix that consist of constants and it is used in order to give certain restrictions to the parameterizing of the conditional variance as response to past residuals. The mean has two parameters: the β which is a vector and δ which is a scalar.

The comparison between the $ARCH-M$ and the $ARCH$ models is almost straightforward from equations (5) and (2).

The three previous models capture some important features of financial time series in an elegant way, especially when it comes to representing the volatility clustering phenomenon. However, there is one aspect in which these models fail, the leverage effect, that is the fact that in financial markets "bad news" contribute more to the increase in volatility than "good news". In terms of financial asset prices, this means that negative surprises have a greater impact on volatility than a positive surprise of the same magnitude (Hamilton, 1994). Empirical evidence shows that the asymmetry in stock markets is a recurrent phenomenon, and as such, a structure that correctly models it is of interest. Due to the way how the $ARCH$ and $GARCH$ models are constructed, none of them are capable to solve this issue, thus Nelson (1991) introduced a new form of $ARCH$ model, termed *exponential* $GARCH$ or $EGARCH$.

To ensure that σ^2 is positive in (2) and (3), nonnegative values to the weights are given in the linear combination. The approach taken by Nelson (1991) to ensure this, is to express $\log(\sigma_t^2)$ as a function of time and lagged $\acute{\varepsilon s}$:

$$\log(\sigma_t^2) = \alpha_t + \sum_{k=1}^{\infty} \beta_k g(\varepsilon_{t-k}), \quad \beta_1 = 1, \tag{6}$$

where $g(\varepsilon_{t-k})$ is a suitable function. The importance of the function $g(\varepsilon_{t-k})$ lies in the fact that it will be responsible for the asymmetry phenomenon described above. In this way, one must assume that it will be a function of both the magnitude and the sign of ε_t. Nelson (1991) proposed the following expression for $g(\varepsilon_{t-k})$

$$g(\varepsilon_t) = \theta\varepsilon_t + \gamma(|\varepsilon_t| - E(\varepsilon_t)),$$

which expresses $g(\varepsilon_t)$ as a linear combination of the magnitude and sign of ε_t. This expression allows the $\{g(\varepsilon_t)\}_{t=-\infty}^{\infty}$ to be a random *iid* sequence with zero mean. Each of the two terms that compose $g(\varepsilon_t)$ have also zero mean. Different combinations of θ and γ allow us to model the way $\log(\sigma_t^2)$ responds to the magnitude of the innovation ε_{t-1} and to its sign, a feature that makes $EGARCH$ more flexible with regards to the asymmetry explained above.

Another popular way to model the asymmetry of positive and negative innovations is the use of indicators functions. Glosten et. al. (1993) presented de $GJR(p,q)$ model

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \left[\gamma_i d(\varepsilon_{t-i} < 0)\varepsilon_{t-i}^2\right] + \sum_{j=1}^{p} \phi_j \sigma_{t-j}^2. \tag{7}$$

where γ_i, for $i = 1, ..., q$, are parameters that have to be estimated, $d(\cdot)$ denotes the indicator function (i.e., $d(\varepsilon_{t-i} < 0) = 1$ if $\varepsilon_{t-i} < 0$ and $d(\varepsilon_{t-i} < 0) = 0$ otherwise). The GJR model allows good news ($\varepsilon_{t-i} > 0$) and bad news ($\varepsilon_{t-i} < 0$) to have differential effects on the conditional variance. For example, in the $GJR(0,1)$ model, good news has an impact of α_1, while bad news has an impact of $\alpha_1 + \gamma_1$. For $\gamma_1 > 0$ the leverage effect exists.

Engle and Ng (1993) recommended the "news impact curve" as a measure of how news are incorporated into volatility estimates by alternative $ARCH$ models. Engle and Ng argued that the GJR model is better than the $EGARCH$ model because the conditional variance implied by the latter is too high due to its exponential functional form.

To be able to predict the volatility for the returns series, one first has to fit one of the parametric volatility models we briefly reviewed. This is done via estimation of the parameters in the model. The most common method is the Maximum Likelihood Estimation. Under this method it is assumed that the data, say $y_1, y_2, ..., y_T$, are random observations drawn independently from the same distribution $F_y(y; \boldsymbol{\theta})$ that depends on the unknown parameter set $\boldsymbol{\theta}$ (for example, in the $GARCH(p,q)$, $\boldsymbol{\theta} = (\boldsymbol{\alpha}_0, \alpha_1, \ldots, \alpha_q, \beta_1, \ldots, \beta_p)'$), so the likelihood function is

$$\pounds(\boldsymbol{\theta}) = f(\boldsymbol{\theta}\,|y_1, y_2, \ldots, y_T) = f(y_1\,|\boldsymbol{\theta})f(y_2\,|\boldsymbol{\theta})\cdots f(y_T\,|\boldsymbol{\theta}) = \prod_{t=1}^{T} f(y_t\,|\boldsymbol{\theta}). \tag{8}$$

Under the assumption that the innovations are *iid* normally distributed, the maximum likelihood estimator of $\boldsymbol{\theta}$ is found by maximizing (8) with respect to $\boldsymbol{\theta}$; however, the likelihood function depends upon the parameters in a highly nonlinear fashion, and frequently numerical optimization techniques are required in order to find the value of $\boldsymbol{\theta}$ that maximizes the likelihood function. Under the assumption of *iid* normally distributed innovations, maximum likelihood estimators are consistent.

In spite of the theoretical properties of maximum likelihood estimators, there are serious bias and inconsistency problems when conditional volatilities are estimated using returns in the presence of microstructure problems. The microstructure problems have been widely analyzed in the context of high-frequency data, where it is found that variations in bid-ask spreads, infrequent transactions, changes in traded volumes, holiday effects, etc., generate big intraday fluctuations in returns that can affect the process characterizing the conditional second moments of returns. Even if the process describing returns is stationary and the conditional volatility model is well specified, these conditions are not enough to guarantee that in the presence of data problems the measurement error associated to the proxy is well behaved, dramatically affecting the consistency of volatility estimations when they are based on returns (absolute or squared). In other words, there is a problem with the proxy used to estimate volatility, instead of the conditional model used to describe the sample data.

Technically, what happens is that the temporal independency of consecutive observations breaks down and the statistical properties of the measurement error ensuring unbiased and consistent estimations are not longer valid.

Under these circumstances, it is more appropriate to use alternative procedures to estimate volatilities. In this sense, there is a renewed interest in the specialized literature in using the range estimator, defined as the difference between the high and low of log prices. As shown by Andersen et. al., range is an efficient estimator of volatility (see also Brandt and Diebold, 2002). At this respect, Chou (2005) proposed the Conditional Autoregressive Range ($CARR$) model, using a $GARCH$ representation to characterize the time behavior of the range volatility. The $CARR(p,q)$ models is specified as

$$
\begin{aligned}
l_t(x) &= \lambda_t(x)\eta_t; \quad \eta_t\,|\Omega_{t-1} \sim f(s,\xi_t) \\
\lambda_t(x) &= \omega + \sum_{i=1}^{q} \alpha_i l_{t-i}(x) + \sum_{j=1}^{p} \beta_i \lambda_{t-j}(x),
\end{aligned} \tag{9}
$$

where $l_t(x) = \max(\log P_t) - \min(\log P_t)$, $\lambda_t(x)$ is the range volatility, and η_t is an error. All the coefficients in the second equation must be positive in order to assure that the matrix of volatilities be positive definite. As in the case of standard volatility models, the condition for the existence of a CARR process is that $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_i < 1$. Engle and Russel (1998) show that under certain regularity conditions this type of models can be estimated by Quasi-Maximium Likelihood procedures, with an exponential distribution.

More recently, Bórquez (2005) proposes a variant of the CARR model, the $\log -CARR(p,q)$ or $LCARR$ model, that is based on the logarithm of range as a volatility proxy. In this specification, the range is modeled as $l_t(x) = \mu \exp(\xi_t(x))$ where $\mu = E\left[l_t(x)|\Omega_{t-1}\right]$ is the conditional expectation of range and $\exp(\xi_t(x))$ is a non-observable error with distribution given by $\Pi(1, a^2\sigma_t^2)$. Notice that $\ln\left[l_t(x)\right] - \ln\mu = \xi_t(x)$, where $\ln\mu = \frac{1}{T}\sum_{t=0}^{T-1} \ln\left[l_t(x)\right]$. Alizadeh et. al. (2002) described the statistical properties for this volatility estimator and found that the logarithm of the range is distributed approximately as a normal. Then, the evolution of $\xi_t(x)$ can be represented as $\xi_t(x) = \chi_t\varepsilon_t$; and $\varepsilon_t\,|\Omega_{t-1} \sim N(0,1)$, so the $GARCH$ part of the $LCARR$ model is

$$
\chi_t^2(x) = \omega + \sum_{i=1}^{q} \alpha_i \xi_{t-i}(x) + \sum_{j=1}^{p} \beta_i \chi_{t-j}^2(x),
$$

Considering the following change of variable $2\ln u_t(x) = \chi_t^2(x)$, we can conveniently express the process in terms of the log-range estimator,

$$
\ln u_t(x) = c + \sum_{i=1}^{q} \alpha_i \ln\left[l_{t-i}(x)\right] + \sum_{j=1}^{p} \beta_i \ln u_{t-j}(x),
$$

where $c = \omega - \alpha_i \sum_{i=1}^{q} \ln\mu$. According to this, the Quasi Maximum Likelihood method provides consistent and also efficient parameter and standard deviation estimates, even in the presence of microstructure effects. But most importantly, because the innovations in the LCARR model are asymptotically distributed as normal, volatility forecasts are optimal in a Mean Squared Error sense.

3 Empirical Methodology and Results

In this section, we first estimate several univariate models for nine stocks traded in the Chilean capital market. Next, under the assumption of constant conditional correlation of Bollerslev (1990) we extend this to the multivariate case. Given that our focus is on microstructure effects and their implications for consistent volatility estimation and forecasting, we use high, medium and low capitalized securities having full presence in the market[2]. Thus, we check for events of zero (squared) return or range to accommodate transactions problems as they are unlikely to occur in less liquid assets, and we treat them accordingly as missing data.

In order to recover missing data, we follow Corsi et. al. (2001), who showed that information delay due to infrequent trading affects the price discovering process, implying that the assumption of independence for consecutive returns no longer holds. It is possible to think such an event as an extreme case of dependence, where the price does not change until new information arrives. For financial stocks, the authors propose to fill the gap with the previous realization of the variable (i.e. return or range) and then filtering with an ARMA(1,1) process.

The sample includes 1,012 observations of closing, maximum and minimum prices from 1/2/2001 to 1/21/2005. Return series are obtained by $100(\log p_t - \log p_{t-1})$ and the range series are computed by $100(\log p_t^{\max} - \log p_t^{\min})$. Exhibit 1 reports summary statistics for the stocks considered. All the return series display strong data problems showing non-stationarity in levels and first differences. Excess kurtosis is a common characteristic as well as positive skewness although two assets are negative skewed. Moreover, all the series show strong autocorrelation as evidenced by very high values of the Portmonteau statistic. Interestingly, the more liquid assets are also the more capitalized ones as they are less affected by zero return events[3]. In the case of the log-range, we can see that series are stationary in levels with only one exception, the less liquid stock. Also note that unconditional mean is higher for less liquid assets. In Exhibit 2 we graph those series.

We estimate for each one of the nine stocks considered, different univariate GARCH representations and the Riskmetrics along with the LCARR model. Results are reported in Exhibit 3. As we can see, with the exception of one stock, all standard errors are lower when the LCARR specification is used compared to the GARCH models. This is related to the fact that the value of the log-likelihood function is systematically lower for the LCARR model, meaning that there is a higher probability of observing the sample at hand. It is also evident that all the return-based models display a high degree of persistence, as shown

[2] All nine stocks reported transactions every day the market was open.

[3] We use the ratio between non-zero return events to total number of transactions as a proxy for liquidity.

by the estimated autoregressive and moving average parameters. Indeed, such a result was expected, as we noted in Exhibit 1 the non-stationarity comes from the autoregressive part of the return processes and not from the level of the series. In effect, the presence of roots outside the unit circle of the lag polynomial corresponding to the AR part indicates that shocks have permanent effects in volatility, which should be captured by a high persistence. This is also confirmed next in a multivariate context by the higher values of portfolio volatility estimates obtained with the IGARCH model[4], that explicitly considers an integrated process. Conditional volatility estimates and kernel graphs for the residuals of the LCARR model are in Exhibits 4 and 5 respectively.

In coherence with the proposed methodology of multivariate volatility estimation, we validate the assumption of constant conditional correlation for the sample studied. A convenient possibility is to look for a change in the correlation matrix for different independent sub-periods using the test developed by Jennrich (1970). Its statistic has an asymptotic chi-square distribution with $p(p-1)$ degrees of freedom, where p is the number of assets included in the correlation estimation (in our example, $p = 2$). Therefore, we consider in-sample constant conditional correlations and test for changes in the parameters between subsequent periods. However, as pointed out by Chesnay and Jondeau (2000), it is also important to formally recognize a breaking day in order to avoid the selection bias problem. To adequately determine the sub-samples, we follow the procedure of Inclán and Tiao (1994) that is based on a CUSUM test. Given that our interest is only on correlations, we do not attempt for a more sophisticated approach needed for an accurate analysis of covariances[5]. The interested reader may refer to Kim et. al. (2000) for an extension of the Inclán and Tiao's method that allows for detecting structural changes on volatility, in the context of a variety of dependent processes including GARCH processes.

In the same line, Bos et. al. (1998) proposed to use non-linear bands to determine the critical p-values. Accordingly, we were able to detect a possible structural break in the portfolio return over observation 700 of the sample corresponding to October 2, 2003 (See Exhibit 6). Five correlations were significantly different, being four of them at the 1% level. Consequently, in the following we use only the first 700 data to run multivariate models.

After compromising the stability of all conditional correlation estimates, we can report our most significant results. It is possible to see in Exhibit 7, that all different MGARCH

[4] On the contrary, the simplest model is RiskMetrics. In this approach the parameters of the volatility are not estimated but are fixed ex-ante. Then, it is unable to capture conditional elements of the volatility process and this explains why the portfolio variance estimations are greatly affected by non-stationarity of the return series when is compared to GARCH processess.

[5] Instead of the Inclán and Tiao´s test is applied in principle to iid variables, Andreou and Ghysels (2002) have studied its properties in Monte Carlo simmulations to find that this procedure is also powerful when strong dependences are considered.

representations and Riskmetrics do not vary importantly in their estimates between them and from the unconditional matrix for less liquid assets, but fail dramatically in capturing high co-movements when they are most expected, and because data problems we know that these results are biased and inconsistent. We also note that correlations are quite low, which is not intuitive as we observe similar price and return paths for some of the stocks evaluated. Thus, after controlling for individual time-varying variances and different conditional specifications including asymmetries and long memory, we were unable to obtain useful results for the entire portfolio, meaning that daily returns -although filtered by an ARMA process- are seriously affected by microstructure problems and should not be used directly as a volatility proxy in this context.

In determining how important the bias is, we provide for a consistent estimation of the correlation matrix using the MLCARR model. Results are straightforward. Correlations are indeed several times higher[6] than those obtained with GARCH models and Riskmetrics. As the bias is stable across time with respect to MLCARR (not shown to save space), we confirm that data problems arise from complex autoregressive non-stationarity in the returns processes. Implications are extremely important for a variety of financial applications and some of them are revised in the next section.

4 Implications for Financial Applications and Conclusions

Asset return volatilities are central to finance, whether in asset pricing, portfolio allocation, or market risk measurement. Thus, our ultimate goal is to contribute to the measurement, modeling and forecasting of time-varying volatility and associated tools, which is indeed a very difficult task in the presence of microstructure effects that frequently emerge in less developed markets. At this respect, we showed that the MLCARR model provides for consistent estimations of volatility and correlations when transactions are infrequent and we also noted that more traditional return-based models dramatically fail in this aim. This seems to be an important issue in local and foreign markets where current industry practice follow, in general, one of two restrictive approaches in risk measurement, historical simulation or Riskmetrics. Of less importance in that circle, although very present in academics, is the use of GARCH models. In this context, the implications of our paper for financial applications are both simple and direct: the use of return-based conditional volatility models provide strongly biased and inconsistent estimates when transactions are infrequent. As a result,

[6] For some stocks, we found correlations can be as high as ten-fold than those estimated by GARCH models.

portfolio selection is misled as risk assessment is severely underestimated by traditional methods.

In this paper we show that problems in capturing conditional characteristics of return series results in severely underestimation of conditional correlations by GARCH type models and Riskmetrics, causing portfolio variances to be lower than otherwise, and this is due to the presence of microstructure effects. We also pointed out that the simplest model -Riskmetrics- is nevertheless the most affected by the data problems. Exhibit 8 shows portfolio frontiers constructed from a 10-days-ahead-forecast, that are drawn in the plane of mean and standard deviation of returns for the different return-based models considered. For the analysis we consider the restriction of non short sale in the period. Therefore, we have a single graphical representation of the conditional behavior of volatility series as they are captured by GARCH models and RiskMetrics.

First, we verify that RiskMetrics is the weakest model showing the lowest portfolio variance. This result is important for portfolio selection and capital adequacy. For example, if a fund manager takes his/her investment decisions on the base of the portfolio frontier generated by RiskMetrics, he/she could systematically be choosing erroneously riskier prospects than would be consistent respect to his/her risk preferences. This is so, because he/she can not discover the actual relation between risk and return associated to the asset, in a fashion similar to the adverse selection problem faced by agents when confront with information asymmetries. The implications for capital adequacy are also related with the lack of information about the portfolio risk, which limits the response capacity of the designed policies to high risk events, precisely when they are most needed.

Second, GARCH representations are affected by the data mostly in their relative ability to capture long memory. For instance, the IGARCH model is the less affected delivering the highest portfolio variance. GARCH performs as well as GJR-GARCH and both do better than EGARCH, which is consistent with previous literature. We note that even when return asymmetries were significant in several of our estimations, they seem to be less important in helping with data problems.

Now, we continue the analysis with the comparison of GARCH and LCARR models only. Exhibit 9 shows that all possible combination among the nine assets are efficient according to the GARCH model, however, MLCARR clearly detect that the range of portfolios with less than 13% are inefficient and they should not be selected. Therefore, consistent estimation of portfolio variances do provide significant information to risk managers, allowing them to recognize assets that erodes diversification benefits.

Even though we have been working with a limited sample of assets, we expect our main results hold for larger portfolios and less restrictions.

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A The GARCH Family

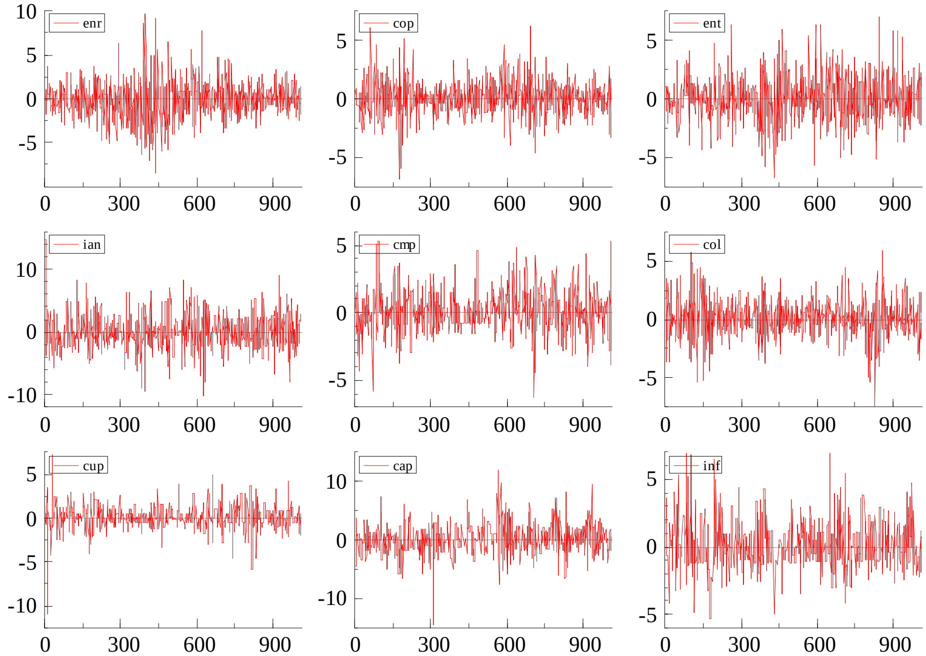
The next table shows some models of the large GARCH family of volatility models.

$ARCH$	$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2$
$GARCH$	$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$
$IGARCH$	$\sigma_t^2 = \omega + \varepsilon_{t-1}^2 + \sum_{i=2}^{p} \alpha_i (\varepsilon_{t-i}^2 - \varepsilon_{t-1}^2) + \sum_{j=1}^{q} \beta_j (\sigma_{t-j}^2 - \varepsilon_{t-1}^2)$
$Taylor/Schwert$	$\sigma_t = \omega + \sum_{i=1}^{p} \alpha_i \left\| \varepsilon_{t-i} \right\| + \sum_{j=1}^{q} \beta_j \sigma_{t-j}$
$AGARCH$	$\sigma_t^2 = \omega + \sum_{i=1}^{p} \left[\alpha_i \varepsilon_{t-i}^2 + \gamma_i \varepsilon_{t-i} \right] + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$
$NAGARCH$	$\sigma_t^2 = \omega + \sum_{i=1}^{p} \left[\alpha_i \varepsilon_{t-i}^2 + \gamma_i \sigma_{t-i} \right]^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$
$VGARCH$	$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \left[\varepsilon_{t-i} + \gamma_i \right]^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$
$TGARCH$	$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \left[(1-\gamma_i)\varepsilon_{t-i}^{+} - (1-\gamma_i)\varepsilon_{t-i}^{-} \right]^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$
$GJR - GARCH$	$\sigma_t^2 = \omega + \sum_{i=1}^{p} \left[\alpha_i + \gamma_i I_{(\varepsilon_{t-i}^2 > 0)} \right] \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$
$\log - GARCH$	$\log(\sigma_t) = \omega + \sum_{i=1}^{p} \alpha_i \left\| \varepsilon_{t-i} \right\| + \sum_{j=1}^{q} \beta_j \log(\sigma_{t-j})$
$EGARCH$	$\log(\sigma_t^2) = \omega + \sum_{i=1}^{p} \left[\alpha_i \varepsilon_{t-i} + \gamma_i (\left\| \varepsilon_{t-i} \right\| - E \left\| \varepsilon_{t-i} \right\|) \right] + \sum_{j=1}^{q} \beta_j \log(\sigma_{t-j})$
$NGARCH$	$\sigma_t^\delta = \omega + \sum_{i=1}^{p} \alpha_i \left\| \varepsilon_{t-i} \right\|^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta$
$APARCH$	$\sigma_t^\delta = \omega + \sum_{i=1}^{p} \alpha_i \left[\left\| \varepsilon_{t-i} \right\| - \gamma_i \varepsilon_{t-i} \right]^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta$
$GQ - ARCH$	$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i} + \sum_{i=1}^{p} \alpha_{ii} \varepsilon_{t-i}^2$ $+ \sum_{i<j}^{p} \alpha_{ij} \varepsilon_{t-i} \varepsilon_{t-j} + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$
$HGARCH$	$\sigma_t^\delta = \omega + \sum_{i=1}^{p} \alpha_i \delta \sigma_{t-j}^\delta \left[\left\| \varepsilon_t - \kappa \right\| - \tau(\varepsilon_t - \kappa) \right]^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$
	$\sigma_t^2 = \begin{cases} \left\| \delta \phi_t - \delta + 1 \right\|^{\frac{1}{\delta}}, & \text{si } \delta \neq 0 \\ \exp(\phi_t - 1), & \text{si } \delta = 0 \end{cases}$
$Aug - GARCH$	$\phi_t = \omega + \sum_{i=1}^{p} \left[\alpha_{1i} \left\| \varepsilon_t - \kappa \right\|^\upsilon + \alpha_{2i} \max(0, \kappa - \varepsilon_{t-i})^\nu \right] \phi_{t-j}$ $+ \sum_{i=1}^{p} \left[\alpha_{3i} f(\left\| \varepsilon_t - \kappa \right\|, \nu) + \alpha_{4i} f(\max(0, \kappa - \varepsilon_{t-i}), \nu) \right] \phi_{t-j}$ $+ \sum_{j=1}^{q} \beta_j \phi_{t-j}^2$ $f(x, \nu) = \frac{x^\upsilon - 1}{\nu}$

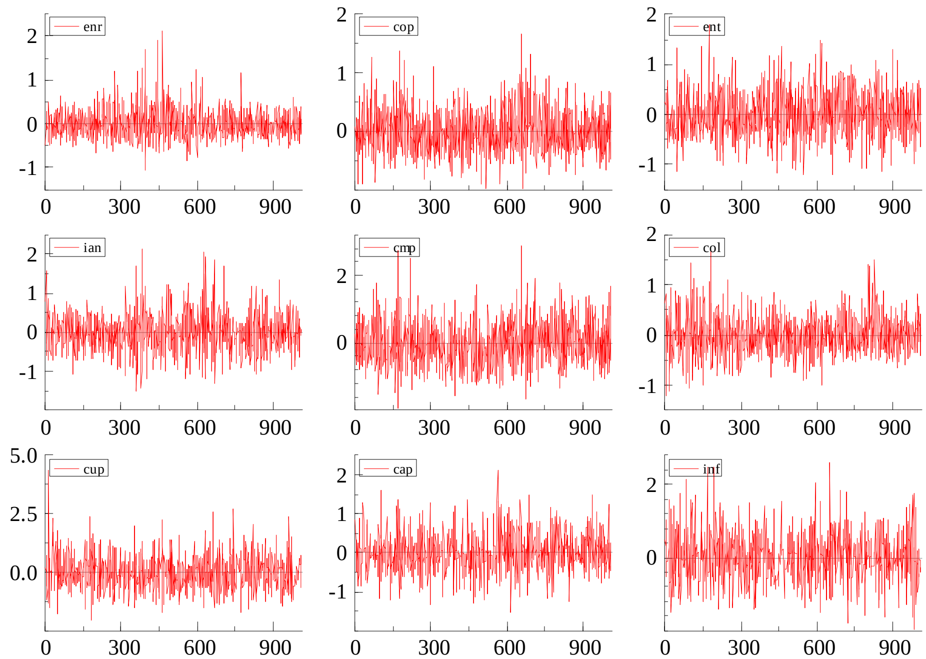
Exhibit 1: Summary Statistics

Returns	MEAN	STDEV.	J-B (p-value)	t-ADF	F-prob	Portmonteau
ENERSIS	-0,0089	1,9692	0,00	-31,70	(0,90)	97,63
COPEC	0,091	1,3395	0,00	-31,71	(0,83)	69,55
ENTEL	-0,003	1,7275	0,00	-31,77	(0,32)	68,49
IANSA	0,002	2,5191	0,00	-32,92	(0,78)	107,68
CMPC	0,088	1,3569	0,00	-31,39	(0,37)	250,22
COLBUN	0,085	1,4995	0,00	-31,90	(0,48)	93,22
CUPRUM	0,159	1,2085	0,00	-31,73	(0,86)	198,13
CAP	0,192	2,2264	0,00	-31,82	(0,73)	234,99
INFORSA	0,114	1,5588	0,00	-31,18	(0,09)	439,7

Log-Range	MEAN	STDEV.	J-B (p-value)	t-ADF	F-prob
COPEC	0,078	0,3888	0,00	-22,10	(0,00)
ENTEL	0,144	0,4651	0,00	-23,76	(0,00)
IANSA	0,138	0,5011	0,00	-22,65	(0,00)
CMPC	0,258	0,655	0,00	-22,52	(0,00)
COLBUN	0,088	0,408	0,00	-21,03	(0,00)
CUPRUM	0,295	0,7716	0,00	-19,42	(0,00)
CAP	0,179	0,5451	0,00	-17,69	(0,00)
INFORSA	0,252	0,6645	0,00	-21,30	(0,32)



Unconditional return series



Unconditional demeaned Log-Range series

Exhibit 2

Exhibit 3: Estimation Results

LCARR	Const.	MA	AR	Uncond. Variance	Log-Likelihood	SBT (p-value)
ENERSIS	0,001 (0,001)	0,047** (0,018)	0,944** (0,025)	1,595	-222,542	0,19
COPEC	0,004 (0,003)	0,019 (0,011)	0,949** (0,000)	1,531	-284,962	0,87
ENTEL	0,133** (0,029)	0,111* (0,056)	0,212 (0,148)	1,974	-418,399	0,05
IANSA	0,026 (0,014)	0,129** (0,042)	0,754** (0,097)	2,05	-441,635	0,17
CMPC	0,057 (0,075)	0,051 (0,033)	0,798** (0,023)	3,568	-935,818	0,6
COLBUN	0,000 (0,000)	0,010 (0,006)	0,986** (0,005)	1,456	-273,082	0,07
CUPRUM	0,177** (0,042)	0,326** (0,081)	0,329** (0,001)	5,012	-701,736	0,02
CAP	0,111** (0,036)	0,194** (0,058)	0,294 (0,190)	2,208	-441,296	0,41
INFORSA	0,205** (0,078)	0,130** (0,052)	0,349* (0,208)	3,644	-658,237	0,76

GARCH	Const.	MA	AR	Uncond. Variance	Log-Likelihood
ENERSIS	0,055* (0,033)	0,079** (0,021)	0,906** (0,027)	3,801	-2014,93
COPEC	0,053* (0,027)	0,094** (0,026)	0,880** (0,034)	2,076	-1666,964
ENTEL	0,467 (0,334)	0,196** (0,072)	0,659** (0,169)	3,239	-1949,062
IANSA	0,599 (0,313)	0,124** (0,029)	0,778** (0,072)	6,093	-2326
CMPC	0,157** (0,058)	0,213** (0,037)	0,072** (0,053)	2,24	-1660,62
COLBUN	0,176 (0,178)	0,219 (0,115)	0,711** (0,188)	2,533	-1742,46
CUPRUM	0,209* (0,086)	0,446** (0,094)	0,493** (0,089)	3,473	-1490,9
CAP	0,483* (0,192)	0,233** (0,063)	0,686** (0,079)	5,958	-2153,72
INFORSA	0,979* (0,313)	0,331** (0,083)	0,288* (0,171)	2,569	-1828,94

IGARCH	Const.	MA	AR	Log-Likelihood
ENERSIS	0,027 (0,017)	0,086** (0,025)	0,914	-2016,59
COPEC	0,028* (0,013)	0,111** (0,030)	0,889	-1669,4
ENTEL	0,149 (0,258)	0,219 (0,250)	0,78	-1957,71
IANSA	0,264* (0,121)	0,184** (0,050)	0,816	-2338,68
CMPC	0,099** (0,031)	0,262** (0,047)	0,738	-1664,33
COLBUN	0,097 (0,052)	0,247** (0,086)	0,753	-1745,98
CUPRUM	0,186** (0,053)	0,500** (0,084)	0,499	-1491,68
CAP	0,359 (0,187)	0,310** (0,103)	0,69	-2158,61
INFORSA	0,805** (0,260)	0,747** (0,163)	0,253	-1847,94

RISKMETRICS	Const.	MA	AR	Log-Likelihood
ENERSIS	0,064 (0,056)	0,06	0,94	-2020,17
COPEC	-0,020 (0,048)	0,06	0,94	-1680,79
ENTEL	0,028 (0,064)	0,06	0,94	-1970,78
IANSA	0,036 (0,095)	0,06	0,94	-2353,37
CMPC	-0,019 (0,049)	0,06	0,94	-1710,31
COLBUN	0,019 (0,048)	0,06	0,94	-1772,87
CUPRUM	-0,026 (0,049)	0,06	0,94	-1561,72
CAP	-0,013 (0,110)	0,06	0,94	-2210,37
INFORSA	0,009 (0,064)	0,06	0,94	-1873,46

GJR-GARCH	Const.	MA	AR	GJR	Log-Likelihood
ENERSIS	0,064* (0,032)	0,057* (0,023)	0,903** (0,026)	0,046* (0,022)	-2013,11
COPEC	0,052 (0,029)	0,078** (0,023)	0,876** (0,034)	0,042 (0,033)	-1665,67
ENTEL	0,434 (0,277)	0,121 (0,064)	0,679** (0,146)	0,134* (0,058)	-1945,3
IANSA	0,531* (0,276)	0,093** (0,032)	0,790** (0,065)	0,064 (0,036)	-2324,54
CMPC	0,159** (0,058)	0,228** (0,048)	0,714** (0,053)	-0,027 (0,055)	-1660,48
COLBUN	0,165 (0,128)	0,186* (0,093)	0,724** (0,140)	0,049 (0,051)	-1741,91
CUPRUM	0,209** (0,081)	0,509** (0,121)	0,497** (0,089)	-0,141 (0,127)	-1489,45
CAP	0,556** (0,189)	0,307** (0,094)	0,664** (0,073)	-0,139 (0,091)	-2149,94
INFORSA	0,968** (0,302)	0,301** (0,109)	0,295 (0,166)	0,054 (0,097)	-1828,74

EGARCH	Const.	MA	AR	EGARCH(1)	EGARCH(2)	Log-Likelihood
ENERSIS	1,231** (0,248)	-0,670** (0,176)	0,989** (0,007)	-0,032 (0,028)	0,303** (0,079)	-2007,98
COPEC	0,612** (0,184)	-0,328 (0,243)	0,953** (0,024)	-0,038 (0,027)	0,316** (0,067)	-1662,62
ENTEL	1,151** (0,139)	-0,598** (0,200)	0,959** (0,029)	-0,081** (0,037)	0,342** (0,066)	-1942,41
IANSA	No convergence					
CMPC	0,583** (0,127)	0,007 (0,215)	0,851** (0,046)	0,03 (0,033)	0,412** (0,071)	-1660,82
COLBUN	0,669** (0,107)	0,188 (0,323)	0,691** (0,132)	-0,019 (0,034)	0,577** (0,071)	-1742,59
CUPRUM	0,381 (0,246)	-0,105 (0,192)	0,836** (0,067)	0,042 (0,041)	0,631** (0,087)	-1482,96
CAP	1,553** (0,140)	0,064 (0,224)	0,792** (0,070)	0,068 (0,044)	0,456** (0,078)	-2160,43
INFORSA	0,892** (0,121)	-0,668** (0,146)	0,917** (0,062)	0,005 (0,041)	0,495** (0,067)	-1827,424

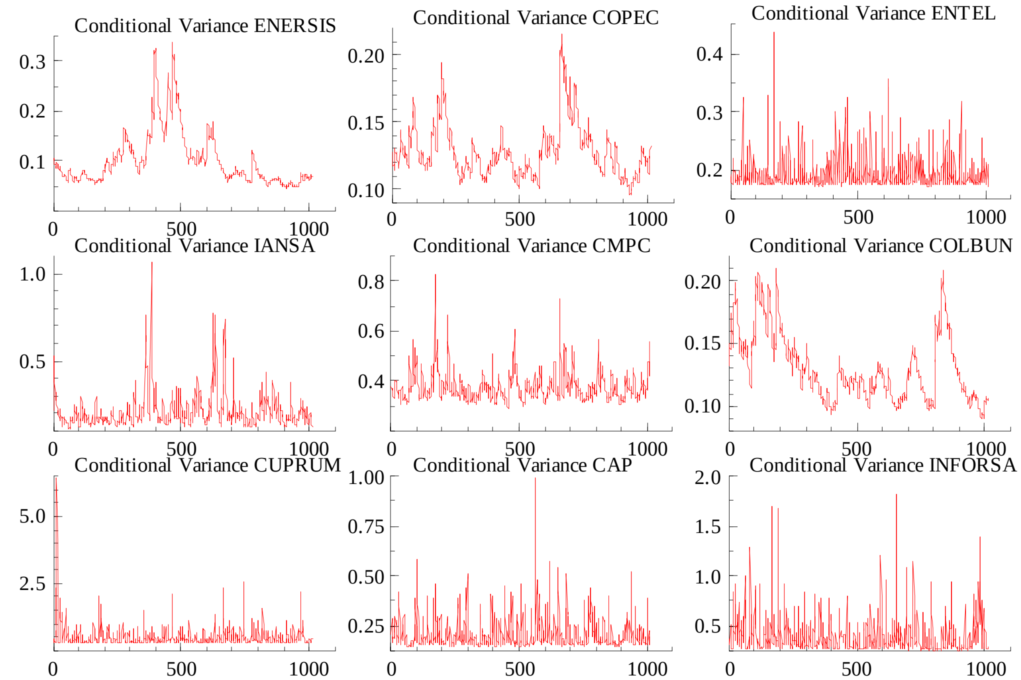


Exhibit 4: Condicional variances (in demeaned logs) for the LCARR model

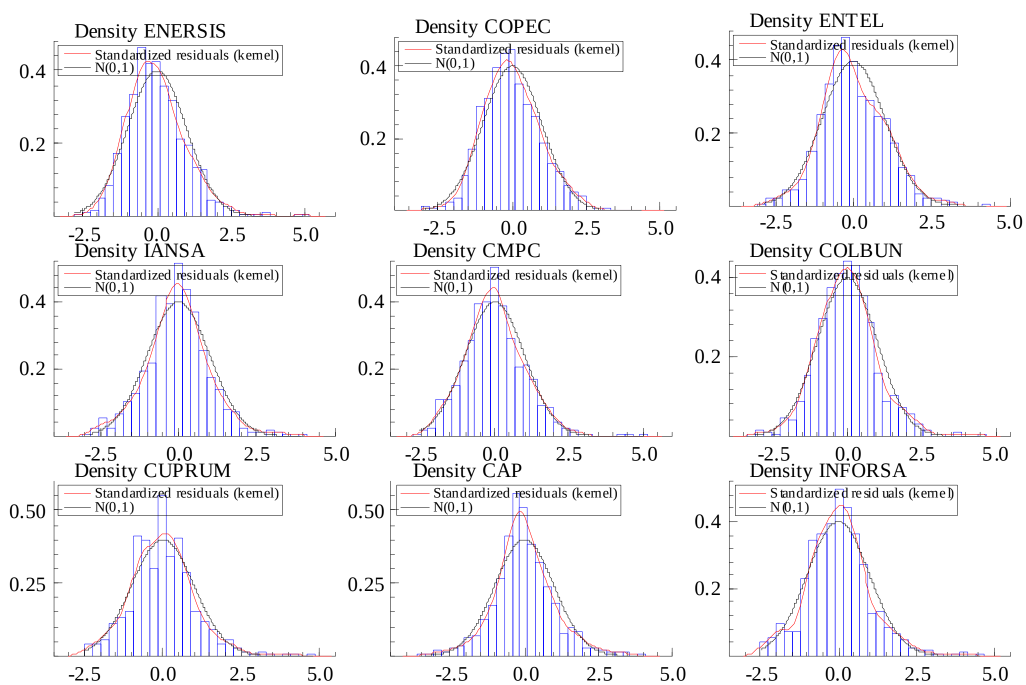


Exhibit 5: Kernel graphs for the residual of the LCARR model

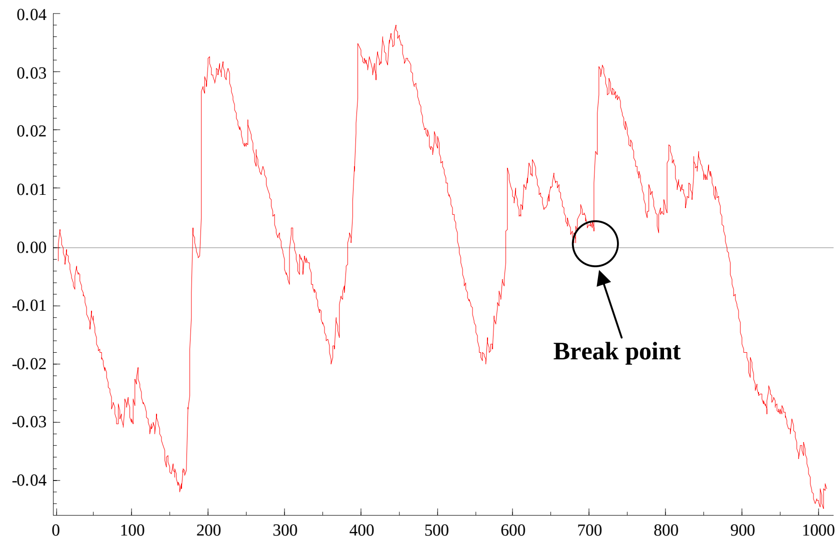


Exhibit 6: Test of Inclán and Tiao (1994) and Bos et.al. (1998)

Exhibit 7

Unconditional	ENERSIS	COPEC	ENTEL	IANSA	CMPC	COLBUN	CUPRUM	CAP	INFORSA
ENERSIS	1.0000								
COPEC	0.28939	1.0000							
ENTEL	0.29486	0.26383	1.0000						
IANSA	0.19118	0.13473	0.13991	1.0000					
CMPC	0.15106	0.35630	0.20918	0.080816	1.0000				
COLBUN	0.29276	0.26388	0.25648	0.18789	0.16028	1.0000			
CUPRUM	0.10412	0.14658	0.11233	0.063044	0.067400	0.050115	1.0000		
CAP	0.17494	0.20556	0.18526	0.093071	0.10037	0.18109	0.078778	1.0000	
INFORSA	0.17098	0.18083	0.12550	0.12626	0.15618	0.12847	0.069457	0.15713	1.0000

GARCH

ENERSIS	1.0000								
COPEC	0.10099	1.0000							
ENTEL	0.21649	0.15586	1.0000						
IANSA	0.20244	0.35398	0.27256	1.0000					
CMPC	0.088053	0.075278	0.094850	0.13521	1.0000				
COLBUN	0.19359	0.17130	0.31862	0.29669	0.10998	1.0000			
CUPRUM	0.20705	0.20614	0.23862	0.24154	0.11182	0.26335	1.0000		
CAP	0.092837	0.090084	0.19901	0.13765	0.070386	0.18639	0.11556	1.0000	
INFORSA	0.16192	0.19691	0.17292	0.19224	0.10460	0.19634	0.13231	0.13258	1.0000

IGARCH

ENERSIS	1.0000								
COPEC	0.090874	1.0000							
ENTEL	0.20132	0.14700	1.0000						
IANSA	0.18783	0.33113	0.25802	1.0000					
CMPC	0.083969	0.070868	0.092016	0.12861	1.0000				
COLBUN	0.17951	0.16054	0.30435	0.27846	0.10630	1.0000			
CUPRUM	0.19508	0.19462	0.22517	0.22934	0.10725	0.25569	1.0000		
CAP	0.075465	0.077765	0.17816	0.12818	0.063571	0.16966	0.10400	1.0000	
INFORSA	0.14575	0.18358	0.16333	0.17867	0.10719	0.18309	0.12577	0.11467	1.0000

GJR-GARCH

ENERSIS	1.0000								
COPEC	0.10270	1.0000							
ENTEL	0.21370	0.15550	1.0000						
IANSA	0.20399	0.35580	0.27109	1.0000					
CMPC	0.089701	0.074498	0.093069	0.13480	1.0000				
COLBUN	0.19316	0.17205	0.32087	0.29690	0.11011	1.0000			
CUPRUM	0.20802	0.20605	0.24125	0.23951	0.11216	0.26068	1.0000		
CAP	0.095716	0.091053	0.19855	0.13533	0.071217	0.18510	0.11515	1.0000	
INFORSA	0.16349	0.19709	0.17389	0.19440	0.10536	0.19550	0.12997	0.13250	1.0000

EGARCH

ENERSIS	1.0000								
COPEC	0.10194	1.0000							
ENTEL	0.20228	0.16255	1.0000						
IANSA	0.20305	0.36105	0.27209	1.0000					
CMPC	0.089425	0.080646	0.092419	0.14170	1.0000				
COLBUN	0.19204	0.17134	0.32021	0.30152	0.11028	1.0000			
CUPRUM	0.20419	0.20838	0.24403	0.24404	0.11602	0.26244	1.0000		
CAP	0.0079287	0.10336	0.21809	0.14496	0.065204	0.18412	0.13374	1.0000	
INFORSA	0.15354	0.19514	0.17371	0.19332	0.095364	0.19675	0.13484	0.15784	1.0000

RISKMETRICS

ENERSIS	1.0000								
COPEC	0.096647	1.0000							
ENTEL	0.23744	0.19225	1.0000						
IANSA	0.22153	0.37639	0.30046	1.0000					
CMPC	0.087547	0.048109	0.086663	0.14907	1.0000				
COLBUN	0.21226	0.17283	0.35166	0.31490	0.11956	1.0000			
CUPRUM	0.22172	0.21778	0.26595	0.25605	0.10387	0.27806	1.0000		
CAP	0.092576	0.083384	0.19769	0.14798	0.074286	0.19129	0.11346	1.0000	
INFORSA	0.18252	0.19140	0.16329	0.19320	0.088905	0.20077	0.12201	0.14882	1.0000

LCARR

ENERSIS	1.0000								
COPEC	0.62023	1.0000							
ENTEL	0.75289	0.65949	1.0000						
IANSA	0.76076	0.70972	0.81085	1.0000					
CMPC	0.56681	0.51191	0.62463	0.60904	1.0000				
COLBUN	0.77301	0.67467	0.82443	0.82960	0.63770	1.0000			
CUPRUM	0.80286	0.72005	0.87456	0.86872	0.68400	0.89018	1.0000		
CAP	0.69775	0.60824	0.74999	0.73953	0.57641	0.77505	0.79294	1.0000	
INFORSA	0.62812	0.56162	0.65352	0.68586	0.50314	0.68149	0.71443	0.61453	1.0000

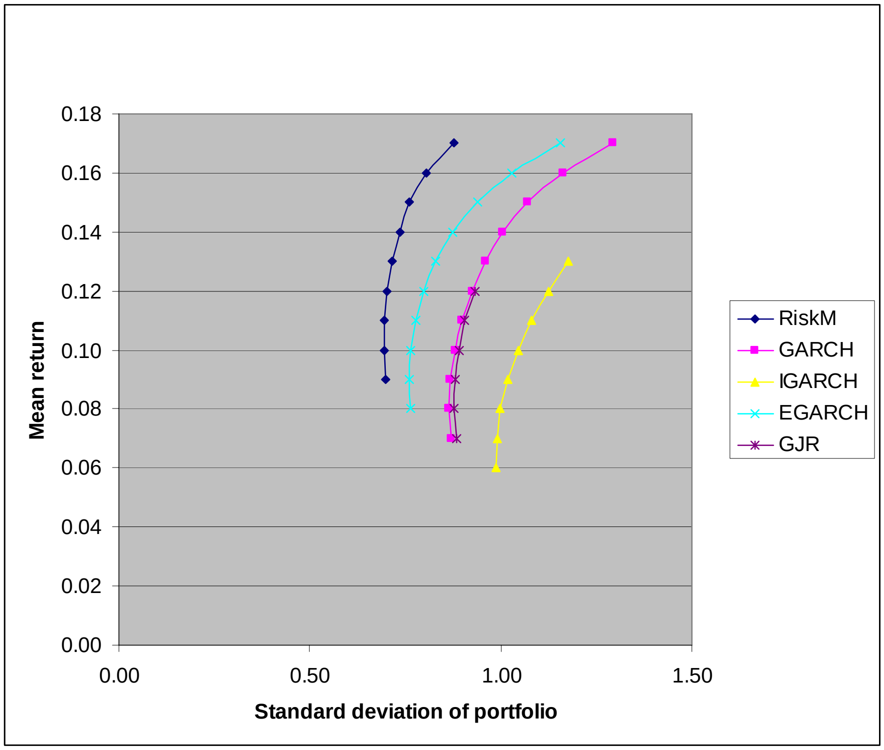


Exhibit 8

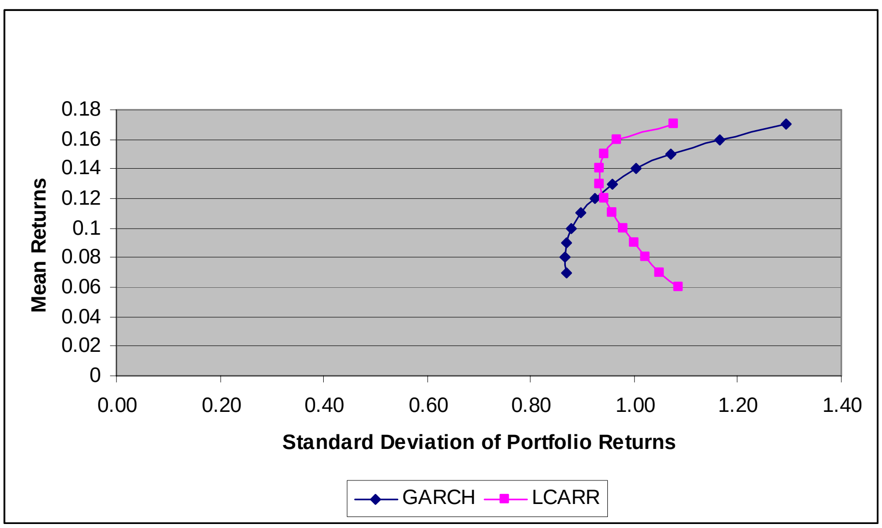


Exhibit 9

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