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A Dynamic Model of Strategic Trade The Case Of Addictive Goods

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A DYNAMIC MODEL OF STRATEGIC TRADE

THE CASE OF ADDICTIVE GOODS

By

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Abstract

We study the impact of dynamic demand on the strategic trade policy. Two two countries export an addictive good to a third country the government of the export firms can use trade policy (subsidies or taxes) to give commitment to its firm to a certain level of production in the steady state. Using the well-known idea of Perfect Markov Equilibrium we find that the optimal trade policy depends on specific values of the structural parameters as well as the consumers' expectation about firms' strategies. Nevertheless when the cost function is restricted to be linear in the production of the good, the optimal trade policy turns out to be a subsidy.

Introduction

Strategic trade theory became popular among trade economists after the seminal paper of Brander and Spencer (1985). One feature of this theory is the sensitivity of the results to different assumptions about demand, about market structure and about the timing of movement between governments and firms. The introduction of dynamic elements, either on the demand or supply side, has been explored recently in this theory. In the work of Driskill and McCafferty (1989) the dynamic element comes from the adjustment cost that two firms, one from the home country and the other from the foreign country, face when firms change production in order to export to a third country. On the other hand, Driskill and Horowitz (1996) study dynamic on the demand side. They analyze the impact of trade policy on exports of a durable good from two firms (home and foreign) to a third country market. Using the notion of Markov Perfect Equilibrium, they find that for the outright sales case the optimal trade policy turns out to be a tax, while for the leasing case the optimal policy is a subsidy. Nevertheless, when modeling dynamic aspect in strategic trade, no one has analyzed the impact of sequential movement between consumers in a third country market and export firms in two countries, one in the home country and the other in the foreign country. My contribution to the dynamic strategic trade theory is to study trade policies using the notion of addiction used in Driskill and McCafferty's (2001) paper on oligopoly provision of addictive goods. The addiction model provides the dynamic aspect in my paper because, for addictive goods, past consumption could affect current consumption. Thus, any decision that firms take about optimal production in an imperfectly competitive market would affect future demand for their goods. This is important because structural parameters affect the optimal trade policy that governments use as a credible commitment tool. Even more importantly, consumers' expectations about firms' strategies could affect the optimal trade policy in equilibrium. When two countries export an addictive good to a third country, the governments of the exporting countries can use trade policy (subsidies or taxes) to ensure commitment of their firms to a certain level of production in the steady state.

Two firms, one from the home country and the other from the foreign country, produce a homogeneous good and sell it in a third country. I refer to the firm in the home country as the home firm and the firm in the foreign country as the foreign firm. First, the governments set the optimal trade policy at the beginning of the game. In the last stage of the game the following subgame is played; first, firms choose repeatedly over time the optimal amount of the good to be produced. In the second stage of the subgame, consumers in the third country choose the optimal amount of consumption. This game is played repeatedly over time. The idea is to construct a Markov Perfect Equilibrium for the subgame. As Karp (1996) notes, without Markov Perfect Equilibrium, a wide variety of possible equilibriums can be supported by trigger strategies. Also, Markov Perfect Equilibrium assumes that the strategies depend on the state variable, which summarizes the whole history of the game. This type of equilibrium has the property of being subgame-perfect. Any unexpected disturbance in the state variable does not change the equilibrium strategy choices of consumers and producers.

In this paper I consider stationary state-dependent strategies for a number of reasons. The infinite horizon models are natural extensions of the finite horizon model when the terminal date tends to infinity. Also, the objective function and the evolution of the state variable do not depend explicitly on time; consequently, the fundamentals of this game remain constant when the length of the game changes. For such strategies it is easier to find the solution for the partial differential equations defined by the first order conditions. However, it is important to restrict the game to specific cases in order to find a solution for the game given the difficulty in analyzing the Markov Perfect Equilibrium. It is well known that linear-quadratic models have linear solutions for the strategies, which makes it easier to study the impact of the trade policy on export of addictive goods.

The methodology described in the previous paragraph allows us to find an explicit solution for the optimal trade policy. However, I find that the optimal trade policy depends on specific values of the structural parameters as well as the consumers' expectations about firms' strategies. Nevertheless, when the cost function is linear in the production of the good, the optimal trade policy turns out to be a subsidy. In the previous chapters, I studied the optimal trade policy when firms choose output or prices simultaneously or sequentially and governments simultaneously choose the trade instrument in the first stage of the game. In this chapter, firms and consumers choose their optimal production and consumption sequentially in a continuous time framework and the governments set their optimal trade policy at the steady state. The finding in the linear cost version of this last model is consistent with one of my previous chapters, wherein firms choose output and the cost functions are linear.

In Section 2, I define and describe the consumer's maximization problem and determine the optimal strategy path and expectations for the consumers. In Section 3, I solve the firm's maximization problem and describe the optimal strategy path for its production. In this section the strategies are restricted to be stationary and state-dependent. In Section 4 I describe the market equilibrium and its conditions. In Section 5, I set up the governments' problems to obtain the optimal trade policy. I also provide some numerical examples. In Section 6, a restricted version of the model is studied, wherein firms have linear cost. Finally, Section 7 summarizes and concludes. All proofs are in the appendixes.

The Consumer problem

To simplify aggregation across consumers, I assume a continuum of identical consumers distributed over an interval of length equal to one. Following Becker and Murphy (1988) and Driskill and McCafferty (2001), the utility function depends on both current and past consumption of the addictive good. The current consumption is denoted by y and the past consumption is captured by:

$$(2.1) \qquad z(t) = \int_{-\infty}^{t} y(\tau) e^{-s(t-\tau)} d\tau \,.$$

Therefore, the law of motion of z(t) is the following linear differential equation; $\dot{z} = y - sz$, wherein the dot over the variable denotes the time derivative and s is the depreciation rate. I also assume that the instantaneous utility function takes the following form:

$$(2.2) \qquad u(y, a, x) = \alpha_0 y - (\alpha/2)y^2 + \beta_0 z - (\beta/2)z^2 + \delta yz + x$$

$$\text{where } \alpha_0, \alpha, \beta_0, \beta \geq 0 \text{ and } \alpha\beta - \delta^2 > 0$$

Consumers at any time take prices as given and choose the consumption of the addictive good (y) and other goods (x) that are different from the addictive good. The terms α_0, α, β_0, β, and δ are parameters. The income flow for the consumers is R, so the instantaneous budget constraint is $py + x = R$, wherein p is the price of the addictive good and the price of other goods is normalized to one. Therefore, the consumer's problem is to maximize the present discounted value of its instantaneous utility:

$$\max_{y} U = \int_{0}^{\infty} u(y(t), z(t), x(t)) e^{-rt} dt.$$

$$(2.3) \qquad \text{s.t.} \; \dot{z} = y - sz.$$

$$py + x = R.$$

Where r denotes consumer's rate of time preferences. The current-value Hamiltonian is

L : $\alpha_0 y - (\alpha/2)y^2 + \beta_0 z - (\beta/2)z^2 + \delta yz + R - py + \lambda(y - sz)$. The first order conditions are:

$$(2.4) \qquad \begin{aligned} &\text{(i)}\ \frac{\partial L}{\partial y} = \alpha_0 - \alpha y + \delta z - p + \lambda = 0. \\ &\text{(ii)}\ \dot{\lambda} - \lambda r = -\beta_0 + \beta z - \delta y + \lambda s. \\ &\text{(iii)}\ \lim_{T\to\infty} \lambda(T)e^{r^* T} = 0 \end{aligned}$$

Time differentiating equation (2.4.i), substituting this into equation (2.4.ii), using equation (2.4.i) to substitute for the costate variable in equation (2.4.ii), and rearranging yields the following:

$$(2.5) \qquad p(t) - \alpha_0 + \alpha y - \delta z = \frac{\beta_0 - (\beta - \delta s)z + \dot{p}(t) + \alpha \dot{y}}{r + s}.$$

Since there is perfect information, consumers know that firms use Markov strategies (stationary state-dependent strategies). This means that consumers infer at any time that firms' decisions about price and output are linear functions of the state variable z. Therefore, they infer that future values for equilibrium price and output are linear function of the state, z.

$$(2.6) \qquad \begin{aligned} p(t) &= H + hz(t). \\ y(t) &= \gamma_0 + \gamma z(t). \end{aligned}$$

Where y(t) is aggregate output. Using the conditions above and the law of motion of the state variable, we determine the instantaneous demand for the addictive good (See Driskill and McCafferty 2001 for further details):

(2.7) $p(t) = \chi_0 - \alpha y + \chi z$

where $\chi = A/(r^* + 2s - \gamma) = \Psi(\gamma)$,

$A = \delta(r^* + 2s) - \beta)$

and $\chi_0 = (\chi\,\gamma_0 + (r^* + s)\alpha_0 - \beta_0)/(r^* + s)$

As we can see from equation 2.7, firms face a downward sloping demand, which shifts when the stock of z changes. As in Becker and Murphy (1988), and Driskill and Mc Cafferty (2001) addiction takes place when $A > 0$. Note that if $\chi < 0$ ($\delta < 0$) demand shifts down as consumers consume more of the good, and if χ is positive (δ is sufficiently positive), more consumption of the good implies more demand in the future. Also, equation (2.7) describes the relationship between (γ_0, χ_0) and (γ, χ), derived from the consumers' optimal consumption. To obtain the equilibrium values for these parameters, as functions of the structural parameters, we need an additional pair of relationships from firms' behavior.

Firm optimization

There are two identical firms, one in each country, home and foreign, which have the following cost structure $C_i = c_0 y_i + (c y_i^2)/2$, i =h, f. These firms export a homogeneous addictive good to a third country market. Each firm plays simultaneously and continuously over time and takes the other firm's strategy and demand (equation (2.7)) as given. Let t_i denote a tax (if $t_i > 0$) or subsidy (if $t_i < 0$) applied by firm i's government to exports. The problem for firm i is to choose the optimal strategy $y_i(z)$ from a strategy space y_i to maximize the present discounted value of profits:

$$\max_{y_i \in S} \prod_i = \int_0^\infty \{p(y,z)y_i - (c_0 + t_i)y_i - \frac{cy^2{}_i}{2}\}e^{-rt}dt.$$

(3.9)
$$subject\ to\ \dot{z} = y - sz.$$
$$p = \chi_0 - \alpha y + \chi z.$$
$$y = y_h + y_f.$$

S_i = {$y_i(z)$ such that $y_i(z)$ is continuous and differentiable in z}

The first order conditions for firm i are:

$$
(3.10)\quad
\begin{aligned}
&(i)\,\chi_0 - 2\alpha y_i + \chi z - \alpha y_j - (c_0 + t_i) + \lambda_i - c y_i = 0. \\
&(ii)\,\dot{\lambda}_i = \lambda_i \left(r + s - \frac{\partial y_j}{\partial z}\right) + \alpha y_i \frac{\partial y_j}{\partial z} - \chi y_i. \\
&(iii)\,\lim_{T\to\infty} \lambda_i(T) e^{-rT} = 0. \\
&j = h, f,\ j \neq i.
\end{aligned}
$$

Recall that in this case λ_i measures the marginal impact of increasing z (through y_i and y_j) on firm i's profits. If $\lambda_i > 0$ then the firm i´s marginal revenue is less than its marginal cost. This implies greater output than the static one-shot Cournot game because there is an intertemporal effect of producing more current output on the demand for the addictive good, as long as δ is sufficiently positive.

Time differentiating equation (3.10.i), substituting the result into equation (3.10.ii) and replacing λ_i in the resulting relationship yields a couple of differential equations (firms' strategies). It is well known that linear-quadratic models admit close-loop strategies that are linear in the state variable. Using this idea we guess that the solution to the firm's strategy has the following linear form $y_i = K_i + k_i z$. Substitute this guess into the differential equation we get a couple of linear equations for K_i and k_i. It turns out that the solutions for k_i yields $k_f = k_h = k(\gamma, \chi)$. Aggregation of these strategies yields the following linear relationship $y = K_f + K_h + 2kz$. In equilibrium, this linear relationship between y and z must be equal to that in equation (2.6). Equating coefficients ($2k = \gamma$ and $\gamma_0 = K_f + K_h$) and solving for χ yield a pair of relationships between (χ,γ) and (χ_0,γ_0) derived from consumer and firm behavior:

$$(3.11)\quad \begin{aligned} &(i)\chi = \frac{\gamma^2(4\alpha+3c)-\gamma(r+2s)(3\alpha+2c)}{2(\gamma-r-2s)} = \Phi(\gamma). \\ &(ii)\gamma_0 = \frac{(2(r+s)-\gamma)(2c_0+t_h+t_f-2\chi_0)}{2\chi+2\alpha(3(\mathrm{r}+\mathrm{s})-4\gamma+c(2(r+s)-3\gamma)}. \end{aligned}$$

where $k_h = k_f = k$, $\gamma = 2k$ and $\gamma_0 = K_h + K_f$.

Notice that from equations (2.6) and (2.7) we have the following equilibrium price:

$$(3.12)\quad p = H + hz$$

where $H = \chi_0 - \alpha\gamma_0$, $h = \chi - \alpha\gamma$.

Notice that $\chi > 0$ when $\gamma > 0$, which means that $A > 0$ and at any time the demand curve facing firms shifts outward when z increases. Also $h = \chi - \alpha\gamma > 0$ so the equilibrium price at any time increases when z increases (see Appendix A for the proof).

Market equilibrium for a given government trade policy

Following the Turnpike properties[1] of the Driskill and MacCafferty model, relationships (2.8) and (3.11) determine the values for the endogenous parameters (χ_0, γ_0, χ, and γ) and thus, the values for H and h. Given these values and the initial value for z, the paths of the endogenous variables are determined. These paths characterize the Markov Perfect Equilibrium for the preceding subgame. Since we are interested in the stable equilibrium, wherein the endogenous variables approach their steady state values for any initial condition, we need to ensure that, in equilibrium $\gamma < s$. To ensure that this inequality is satisfied, we impose the following condition (see Appendix A for the proof): $\Phi(s) > \Psi(s)$:

$$\text{(4.13)} \qquad \frac{A}{r+s} < \frac{s(\alpha(3r+2s)+2c(2r+s))}{2(r+s)}.$$

If we assume this condition then there exists a unique stable Markov Perfect Equilibrium of the preceding game. This equilibrium is described by consumers' behavior, firms' optimization, and parameters' relationships, given any pair of t_i (i = h, f). Since our interest is in the stable steady state equilibrium, we set all the time derivatives equal to zero and use all the first order conditions to get the following steady state values:

Firm strategies:

(i) $\bar{y}_i = K_i + k\bar{z}$, $i = h, f$, $K_h + K_f = \gamma_0$, $k = \gamma/2$

(ii) $\chi = \Phi(\gamma) = \Psi(\gamma)$

Reduced form for output, profits and price function (see appendix B for the proofs):

(iii) $p = H + hz$

(iv) $\bar{z} = \dfrac{(\bar{y}_h + \bar{y}_f)}{s}$

(v) $\bar{p} = \alpha_0 + \beta/(r + s) - \bar{y}\,\{(\alpha s(r+s) - A)/(s(r + s))\}$, $A = \delta(r^* + 2s) - \beta < \alpha s(r+s)$,

$\bar{y} = \bar{y}_h + \bar{y}_f$

(vi) $\bar{y}_i = (\gamma - 2(r + s))(2s\chi(\alpha_0(r + s) + ß_0 - (c_0 + t_1)(r + s)) + A(\gamma - 2(r + s))(t_i - t_j) - \alpha s(r + s)(2\alpha_0(r + s) + 2ß_0 - 2c_0(r + s) + \gamma(t_i - t_j) + 2(r + s)(t_j - 2t_i)))/$

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[1] Driskill and MacCafferty (200), and, Fershtman and Morton (1990) develop a turnpike result in which the infinite-horizon equilibrium path for the endogenous variables is chosen among all the equilibriums in a manner that it is the closest one to the equilibrium for the finite-horizon game when the length of the horizon goes to infinite.

$(4(s\chi^2(r + s) - \chi(A(\gamma - 2(r + s)) + \alpha s(4(r + s) - \gamma)(r + s)) + \alpha(r + s)(A(\gamma - 2(r + s)) + \alpha s(3(r + s) - \gamma)(r + s))))$. $i, j = h, f, i \neq j$.

(vii) $$\overline{\Pi}_i = (\bar{p} - (c_0 + t_i) - c\frac{\bar{y}_i}{2})\bar{y}_i$$

Where "-" denotes steady state value for the equilibrium of the subgame.

Now, we are able to analyze the impact of the government policy.

Optimal trade policy

In this case we only take into account the optimal trade policy for the steady state equilibrium. One can consider more general rules, such as $t_i(z)$, but this type of rule tends to excessively complicate the model without adding new insights. Therefore, the government wants to maximize the national welfare at the steady state choosing the optimal trade instrument t_i given t_j. National welfare at the steady state for the home country is defined as:

$$W_h(t_h, t_f) = (\bar{p} - c_0 - \frac{c\bar{y}_h}{2})\bar{y}_h(t_h, t_f). \tag{5.14}$$

Similarly, the national welfare at the steady state for the foreign country is:

$$W_f(t_h, t_f) = (\bar{p} - c_0 - \frac{c\bar{y}_f}{2})\bar{y}_f(t_h, t_f). \tag{5.15}$$

The analytic solution for the governments' maximization problem is reported in the appendix C. Nevertheless, there is no clear optimal trade policy because of the complexity of the general solution. More specific, the trade policy for this model turns out to depend on certain values of the structural parameters. However, the use of numerical simulation using a wide variety of parameter values is a common practice to analyze the nature of the steady state equilibrium in solving problems involving differential games as Dinopoulos (1988) noted[2]. Table 1 displays numerical calculations for the optimal trade policy, given various values for the parameters.

Table 22. Optimal trade policy for various values for the structural parameters

R	S	A	α	α_0	β_0	c0	C	t_i
0.1	0.7	0.5	10000	0.5	0.5	0.1	100	Subsidy
0.1	0.7	0.5	10000	0.5	0.5	0.1	200	Subsidy
0.1	0.7	1	10000	0.5	0.5	0.1	200	Subsidy
0.1	0.7	1	10000	0.5	0.5	0.1	0	Subsidy
0.1	0.7	1	0	0.5	0.5	0.1	0	Subsidy
0.5	0.5	-2	0	0.5	0.5	0.1	0	Subsidy
0.7	0.7	-1	3	4	4	1	400	Tax
0.1	0.7	-0.5	3	0.5	0.5	0.1	200	Tax
0.7	0.7	-0,6	3	0.5	0.5	0.1	200	Tax

In contrast to the Brander-Spencer model where the optimal trade policy is always a subsidy, there is no clear answer to the optimal trade instrument, as we can see from Table 1 even when there is addiction ($A > 0$). However, subsidy is a robust outcome for the trade policy when c (the parameter for the quadratic expression in the cost function) is zero; that is when the cost function is linear in the production of the good. In the following section we study the trade policy for a specific case: a linear cost function.

A restricted version of the model: the case of a linear cost function

[2] See Driskill and McCafferty (1988) for a specific example.

In this section we consider the particular case of a linear cost function but everything else remains the same. Therefore, the cost structure for any firm is $C_i = c_0 y_i$, i = h, f, which simplifies most of the results found in previous sections. The objective function for the home and foreign government becomes:

$$(6.16) \qquad W_i(t_i, t_j) = (\bar{p} - c_0)\bar{y}_i(t_i, t_j)$$

The optimization problem yields the following solution:

$$(6.17) \qquad t_i = 2s^2(\chi - \alpha(r+s))^2(\alpha_0(r + s) + ß_0 - c_0(r + s))/$$
$$((A - \alpha s(r + s)(\gamma - 2(r + s))(3s\chi(r + s) + A(2(r + s) - \gamma) + \alpha s(\gamma - 5(r + s))(r + s)).$$

Notice that the marginal cost term (in this particular case, c_0) disappears from the denominator, which simplifies the main result. It is straightforward to prove that the denominator is negative when $A < 0$ ($\chi < 0$ and $\lambda < 0$). Thus, the optimal policy is a subsidy on exports because the marginal revenue is greater than the marginal cost. Subsequently, when a firm increases output at the steady state, the marginal revenue for the other firm falls ($A < 0$). The result is a profit-shifting from one firm to the firm whose government has subsidized. As in the Brander-Spencer model, the government can improve national welfare by subsidizing exports. Subsidizing exports is optimal because it ensures firm commitment to more production, while average cost remains the same. When $A > 0$ ($\chi > 0$ and $\lambda > 0$), the denominator is again negative (see Appendix C for the proof) and the optimal policy is still a subsidy. The explanation is the following: since λ measures the marginal impact of increasing z (through y) on the firm's profits in the steady state, the marginal revenue is less than the marginal cost when $\lambda > 0$, which implies $\chi > 0$ and $A > 0$ (see Appendix B for the proof). As a result, any increase in production, through subsidizing one firm, will increase marginal revenue and force the other firm to reduce production, which in turn reduces cost smoothly, because the marginal cost is constant and hence, the reduction in the total cost is proportional to the rate of decline in production. Therefore, profits have been shifted from one firm to the firm that has been subsidized; thus, for both governments the best response is to subsidize exports at the steady state.

Conclusion

The idea for this paper is to study the optimal trade policy in a dynamic setting. Two countries, one in the home country and the other in the foreign country, compete in a third-country market exporting a homogeneous addictive good. In the first stage of the game, governments choose the optimal trade policy for one time that maximizes firms' profits minus the cost of the trade policy at the steady state. In the second stage of the game, consumers and producers sequentially choose their strategies in continuous time. Using a quadratic cost structure and the notion of Markov Perfect Equilibrium we conclude that the optimal trade policy could be either a tax or a subsidy on exports, depending on the specific values for the structural parameters. When we restrict the model to the case of a linear cost function, the optimal trade policy turns out to be a subsidy, regardless of the nature of the good ($A > 0$ or $A < 0$). The reason for the existence of this optimal policy is the fact that it is the only way that governments can shift profits from one firm to another. In other words, by imposing export subsidies the governments can credible commit their firm to an specific level of production (different from the free trade equilibrium) at the steady state and that policy can increase profits.

APPENDIX

(i) Proof for the unique stable equilibrium condition:

We know that the value for γ comes from the equality $\Phi(\gamma) = \Psi(\gamma)$.

$\Phi'(\gamma) = [\alpha(2\gamma - r - 2s)(2\gamma - 3(r + 2s)) + c(3\gamma^2 - 6\gamma(r+s) + 2(r+s)^2]/(2(\gamma - r - 2s) > 0$ when $\gamma < s$ with $\Phi''(\gamma) = [(\alpha + c)(r + 2s)^2]/[\gamma - r - 2s]^3 < 0$, which means $\Phi(\gamma)$ is strictly concave and $\Phi(0) = 0$. The attributes of $\Psi(\gamma)$ depend on the sign of A. When A is negative and for $\gamma < 2s + r$, $\Psi(\gamma)$ is strictly concave ($\Psi''(\gamma) = [-2A]/[\gamma - r - 2s]^3 < 0$ (when $A < 0$) with $\Psi'(\gamma) = A/[\gamma - r - 2s]^2 < 0$ and $\Psi''(0) < 0$). Therefore, there exists a unique solution $\gamma^* < 0$, which also implies $\chi < 0$. When A is positive, $\Psi(\gamma)$ is convex, with $\Psi'(\gamma) > 0$, $\Psi(0) > 0$, and $\Psi(s) = A/(r + s)$. To guarantee that there is a unique solution, we need $\Psi(s) < \Phi(s)$, or $A/(s + r) < \{s\alpha(3r + 2s)\}/(s + r)$.

(ii) Proof that $h = \chi - \alpha\gamma$ is positive:

Recall that $\chi = \dfrac{\gamma^2 4\alpha - \gamma(r + 2s)(3\alpha)}{2(\gamma - r - 2s)}$. Therefore, $\chi - \alpha\gamma = \dfrac{\alpha\gamma(2\gamma - r - 2s)}{2(\gamma - r - 2s)}$

Stability conditions require $\gamma < 0$ and hence, both numerator and denominator are negative. Thus, $\chi - \alpha\gamma > 0$.

To obtain the steady state value for p, set all the time derivatives in equation 2.5 equal to zero. The steady state value for y_i comes from setting all the time derivatives in equation 2.10 equal to zero, using equation 2.5 (after setting its time derivatives equal to zero), substituting the relationship $z = (y_1 + y_2)/s$ into 2.10(i), and solving the system of equation.

(iii) Proof that $\lambda_i > 0$ when $\chi > 0$ and $\lambda_i < 0$ when $\chi < 0$ in the steady state:

In the steady state $\dot{\lambda} = 0$, using equation 2.10(ii) and after substituting k = $\gamma/2$ we have $\lambda_i(r + s - \frac{\gamma}{2}) + \alpha y_i \frac{\gamma}{2} - \chi y_i = 0$. Solving for λ_i we have $\lambda_i = \frac{(2\chi - a\gamma)y_i}{2r + 2s - \gamma}$. From appendix A we know that $2\chi - \alpha\gamma > 0$ when $\chi > 0$ and $2\chi - \alpha\gamma < 0$ when $\chi < 0$. Since the denominator is positive (stability condition), then $\lambda_i > 0$ when $\chi > 0$ and $\lambda_i < 0$ when $\chi < 0$. Because $\chi = A/(r^* + 2s - \gamma)$ then by transitivity $\lambda_i > 0$ when $A > 0$ and $\lambda_i < 0$ when

$A < 0$.

(iv) The optimal trade policy, general case.

Differentiating $W_i(t_h, t_f)$ with respect to t_i, equalizing the result to zero, and invoking symmetry ($t_h = t_f$), yields the optimal tax-cum subsidy:

$t_i = s(4s\chi^2(r + s) - 2s\chi(r + s)(4\alpha(r + s) + c(2(r + s) - \gamma)) + Ac(\gamma - 2(r + s))^2 + \alpha s(r + s)(4\alpha(r + s)^2 + c\gamma(2(r + s) - \gamma)))(\alpha_0(r + s) + ß_0 - c_0(r + s))/(\gamma - 2(r + s))(2s\chi(r + s)(3A - s(r + s)(3\alpha + c)) + 2A(2(r + s) - \gamma) + s(r + s)(2A(\alpha(2\gamma - 7(r + s)) + 2c(\gamma - 2(r + s))) - s(r + s)(2\alpha^2(\gamma - 5(r + s)) + 2\alpha c(2\gamma - 5(r + s)) + c^2(\gamma - 2(r + s)))))$

(v) Proof that the optimal export policy is a subsidy for the linear case.

For this proof it is enough to prove that $t_i < 0$. The numerator is $2s^2(\chi - \alpha(r+s))^2(\alpha_0(r + s) + ß_0 - c_0(r + s))$ which is positive because $(\alpha_0(r + s) + ß_0 - c_0(r + s)) < 0$ (see Driskill and McCafferty (2001) for this to hold). The denominator is a combination of three expressions; $f_1 = A - \alpha s(r + s)$, $f_2 = \gamma - 2(r + s)$ and $f_3 = (3s\chi(r + s) + A(2(r + s) - \gamma) + \alpha s(\gamma - 5(r + s))(r + s))$. The expression $f_1 < 0$ is applicable because of the downward sloping demand function at the steady state. The expression $f_2 < 0$ is necessary because stability requires $\gamma < s$, and the expression $f_3 < 0$ pertains because $\alpha s(r + s)(2(r + s) - \gamma) > A(2(r + s) - \gamma)$ since $f_1 < 0$ and $3s\chi(r + s) < 3\alpha s(r + s)$ because $3s\chi(r + s) < 3\chi(r + s) < 3A(r + s) < 3\alpha s(r + s)^2$. Since $3\alpha s(r + s)^2 + \alpha s(r +$

s)(2(r + s) – $\gamma = \alpha s(5(r + s) - \gamma)(r + s)$ then $3s\chi(r + s) + A(2(r + s) - \gamma) - \alpha s(5(r + s) - \gamma)(r + s) < 0$. Therefore, $f_1 f_2 f_3 < 0$ and hence, $t_i < 0$.

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