Alameda 3363
Estación Central-Santiago
Tel. +56 2 7180765
www.usach.cl

Universidad de Santiago de Chile

Departamento de Economía

Working Paper Series

A Two-Stage Model of R&D with Endogenous Timing in Quantity Competition

Autores: Leonardo J. Basso y Pedro Jara-Moroni

WP 2011 - Nº 02

A Two-Stage Model of R&D with Endogenous Timing in Quantity Competition

Leonardo J. Basso * Pedro Jara-Moroni †

This draft August, 2010
Preliminary draft – please do not quote

Abstract

There is a growing literature that aims at endogenizing the first mover in oligopoly models. Some of these articles have shown that, when market competition is in quantities, the most efficient firm –i.e. the one with smallest marginal cost– will endogenously emerge as a Stackelberg leader. In this paper we show that if firms know that market leadership depends on cost structures, then this affects the way in which firms invest in process R&D to decrease costs, making them more aggressive in seeking cheaper technologies. This is caused by the *hyper-strategic* effect that now R&D investment has, as it not only increases efficiency but changes the mode of competition by creating market leadership. We show that in the vast majority of the parameter space, firms invest more in R&D than when market competition is exogenously simultaneous and, in fact, R&D investments are weekly larger than the First-Best ones. These larger R&D investments, that lead to decreased costs of production, while beneficial for the consumers, may in some cases hurt the firms enough to actually diminish social welfare as compared to the simultaneous market competition case.

Keywords: *R&D, Endogenous timing, Cournot, Stackelberg. JEL: L11, L13, D24.*

* *Civil Engineering Department, Universidad de Chile; lbasso@ing.uchile.cl*
† *Department of Economics, Universidad de Santiago de Chile; pedro.jara@usach.cl*

Cuando hay juegos en dos etapas (inversion, market competition), la literatura usualmente asume que el market competition game tiene una estructura conocida (juego simultaneo o secuencial, cantidades o precios). En estos casos, es facil hoy saber que va a pasar cuando uno mira el juego completo en dos etapas (Fudenberg y Tirole) ==¿ efecto estrategico. Sin embargo, literatura en endougenous timing =¿ muchas veces son diferencias en los costos los que endogeinizan el timing (tanto para Betrand como para cournot, y en varios tipos diferentes de no se que). En ese caso, es evidente que si la inversion es en costos, las firmas van a tomar en cuenta el efecto 'hiper-estrategico' de que cambian el modo delo market competition. Nosotros ilustramos este efecto hiperestrategico tomando como punto de inicio a Jean Claude Van demme.

De hecho, nuestro insight de efecto hiper-estratgico aplica a cualquier variable que define el timing del juego que juegan las firmas, independiente de que sea esto: puede ser costos - cantidad, costos - precio (citas varias), costos - location y cost - precio/quality (mesa y tombak, papers citados en emails), capacidad - precio (Kovenack y alguien ms), and so on.

EN este artculo, agregar:

1. condicin sobre los parametros para unicidad/multiplicidad de equilibrio en estrategis puritanas

2. inversiones con exogenous leadership

3. comparar inversiones ET con inversiones SW Second best (i.e. las mejores inversiones dado market power en 2nd stage)

Spin offs follow ups:

1. Stochastic R&D: la inversion lleva a una distribucion de costos marginales (reduce los costos mucho o poco con diferentes probabilidades). Notar que el subjuego ocurre con costos conocidos, por lo que Van Demme y Hurkesn sigue aplicando.

2. R&D cooperation. Comparar con la literatura: si las firmas cooperan en inversiones, pueden decidir de antemano que haya un lider y un follower?

1. INTRODUCTION

When firms compete in quantities, two modes of competition are traditional: the Cournot model, in which firm simultaneously choose their levels of production, and the Stackelberg model, where one firm choose first, and the other, having observed this, reacts. The results are well-known: a Stackelberg leader achieves higher profits than a Cournot duopolist. But what this model dos not answer, is how a leader is selected: since each firm will strive to obtain the most favorable position for itself, which of the two duopolists will gain victory and obtain this leadership position?

Regarding this point, there is a growing literature that aims at endogenizing the first mover in oligopoly models. Saloner (1987) considers a model in which two periods of production are allowed. Firms simultaneously choose quantities in the first period; these become common knowledge and then firms simultaneously decide how much more to produce in the second period before the market clears. The result is that any outcome on the outer envelope of the two reaction functions lying in between the two Stackelberg outcomes can be sustained as a subgame perfect equilibrium. Hamilton and Slutsky (1990) consider a two-stage action commitment game, with the following rules: each duopolist chooses quantity in one of two periods; choices are simultaneous in each period, but if one player chooses to move early while the other moves late, the latter observes the first-mover's choice. Hence, moving early is profitable if one is the only player to do so, but it is costly if the other commits as well as they may both end-up committing to Stackelberg outputs. The equilibria of this game are multiple, including both the Cournot and the Stackelberg outcomes of the underlying duopoly game, ye the Stackelberg outcome are the only pure undominated equilibria of the game. Finally, the most relevant paper four our purposes is that of van Damme and Hurkens (1999) who consider a quantity setting duopoly game with linear demand and constant marginal costs, but where one firm is more efficient, i.e., has lower marginal cost, than the other. They use the two-stage action commitment game from Hamilton and Slutsky (1990) but select the solution of the game by using the risk-dominance concept from Harsanyi and Selten. (1988). Risk considerations show that committing is less risky for the firm that has the lower marginal cost and therefore, the neutral focal point is the equilibrium in which the low cost firm moves first. In simple words, when in the market competition firms may decide to produce or wait, the more efficient firm endogenously emerges a Stackelberg leader. When both firms are equally efficient, the equilibrium is in mixed strategies, yet the conjecture is that the payoff for each firm will be in between that of a Stackelberg follower and a Stackelberg leader.[1]

In our view, if one takes the idea of firms striving to obtain the most favorable position for themselves, the obvious next question is, how can firms ensure that, in the market competition game, their leadership emerges endogenously? If one takes the results of van Damme and Hurkens as true, that is, that cost structures have an influence on market leadership, then firms will take this into account when deciding on their technology, trying to affect their marginal costs in order to gain the position of the most efficient firm, ensuring their leadership in the market or inversely, may decide not to invest at all in technology and just take the follower's position. And this, clearly puts us in the world of process R&D. This type of R&D is the one that firms undertake in order to improve their production process diminishing their marginal costs. Other types of R&D that are studied in the literature are product R&D (to invest to generate new products or differentiate a firm's product from the rest), and patent races, where firms compete for obtaining a major breakthrough, with the hope to recoup investment costs plus a wide margin under the protection of a patent.

Our research idea then is to look at process R&D as a mean not only to decrease marginal cost, but also as a mean to generate market leadership. In other words, the returns to investment process and R&D are not only efficiency advantages but also a change in the mode of competition. For this, we consider, a three-stage model where firms first decide their R&D investments, which define their cost functions, and then play the two-stage action commitment game as analyzed by van Damme and Hurkens. Cost differences after the R&D game endogenously induce sequential competition in the market, where the most efficient firm acts as leader in quantity competition. Our model results show that if firms know that the timing in the market competition will emerge endogenously depending on their efficiency, then R&D investments are going to be larger than if the market game is simultaneous and might even be larger than first-best R&D investments. In this sense, we find a *hyper-strategic* effect that goes beyond the strategic role for process R&D that Brander and Spencer (1983) found; they showed that, with quantity competition, for strategic reasons, firms would invest more in R&D when investments are prior to output decisions than when these decisions are simultaneous, something known today as a top-dog strategy (Fudenberg, 1984).

[1]Endogenous timing in oligopoly games has been analyzed as well in, among others, Kambhu (1984); Sadanand and Green (1991); Spencer and Brander (1992); Mailath (1993); Sadanand and Sadanand (1996); Amir and Grilo (1999); van Damme and Hurkens (2004) and Amir and Stepanova (2006)

Other important papers related to R&D investments are D'Aspremont and Jacquemin (1988), who include the role of R&D spillovers –something that we also consider– and compare non-cooperative and cooperative games, Suzumura (1992), who considers second best outcomes and n firms, Qiu (1997) who compares the effect of R&D on quantity and price market competition, and Amir and Wooders (2000), Amir et al. (2003) and Atallah (2005) who consider different ways of incorporating the spillovers. The plan of the paper is as follows: in Section 2. we lay out our model and assumptions; in Section 3. we solve for equilibria of the endogenous timing subgame; in Section 4. we solve the (exogenously) simultaneous subgame and the first-best in order to perform comparisons of the different results, something that we undertake in Section 5.. Section 6. concludes.

2. The Model

Our model is similar to D'Aspremont and Jacquemin (1988) and Qiu (1997). We consider a homogenous goods duopoly with linear demand and constant marginal costs, which can be reduced via a pre-market competition investment. The demand function is given by $P = a - q_i - q_j$, where q_i and q_j represent output. Marginal costs are given by $C_i(x_i, x_j) = \bar{c} + c - x_i - \theta x_j$, where x_i represents firm *i's* R&D investment, $\theta \in [0, 1]$ is R&D spillover and $\bar{c}$ represent the minimum marginal cost achievable through R&D. Without lost of generality we set $\bar{c} = 0$. R&D investment has a cost given by $I(x) = \frac{1}{2}vx^2$, and therefore, a firm's payoff is given by

$$\pi_i(x_i, x_j, q_i, q_j) = \big(P - C_i(x_i, x_j)\big)\, q_i - I(x_i) = \big(a - c + x_i + \theta x_j - q_i - q_j\big)\, q_i - vx_i^2 \qquad (1)$$

The timing of the game is as follows: First, firms invest simultaneously (or without observation from the part of the competitor) in R&D. Second, investments and therefore marginal costs are revealed. Then, firms play the two-stage commitment game of Hamilton and Slutsky (1990) and the most efficient firm will endogenously emerge a Stackelberg leader, following van Damme and Hurkens (1999). We focus on the first-stage reduced R&D game only, where payoffs are given by equilibrium market profits minus R&D costs. Market profits are obtained from the Stackelberg competition model with constant marginal costs. Reduced profits for the Stackelberg competition case are then given by:

$$\pi_L(x_L, x_F) = \frac{(a - c - (1 - 2\theta)x_F + (2 - \theta)x_L)^2}{8} - \frac{vx_L^2}{2} \tag{2}$$

$$\pi_F(x_L, x_F) = \frac{(a - c - (3 - 2\theta)x_F + (2 - 3\theta)x_L)^2}{16} - \frac{vx_F^2}{2} \tag{3}$$

where L and F stand for Leader and Follower respectively.

When firms end up with the same marginal cost after R&D investments, van Damme and Hurkens (1999) do not provide an explicit solution of the game. What they show though is that the equilibrium is symmetric and in mixed strategies. For our purposes, what we actually need are the payoff functions in the subgame rather than the explicit solution and, what we now, is that these payoffs are in between the payoff of a Stackelberg follower and Stackelberg leader (van Damme and Hurkens, 1999). For simplicity, and to enhance comparability to the exogenously simultaneous game, we will impose that when firms have the same marginal costs, the payoff they receive in the two-stage commitment game is the Cournot payoff. This payoff is indeed in between that of the follower and the leader of the Stackelberg game, but may not be exactly what should emerge. Thus, in the case of identical marginal cost when timing is endogenous, or exogenous simultaneous game (name that we will continue to use in what follows), the reduced profits are:

$$\pi_S(x_i, x_j) = \frac{(a - c - (2 - \theta)x_i + (1 - 2\theta)x_j)^2}{9} - \frac{vx_i^2}{2} \tag{4}$$

To find pure-strategy equilibria points we perform a unilateral deviation analysis, that is, we consider all points (x_i, x_j) in the strategy space and see whether (at least) one firm has an incentive to deviate from it. For this, it is first convenient to divide the strategy space in different regions, which are represented in Figure 1.

The figure shows four relevant zones. If investments fall on any point of the diagonal between the origin and point $A = (c/(1+\theta), c/(1+\theta))$, this implies that firms invest the same, and therefore end up with identical marginal costs. The continuation game then will be, endogenously, a simultaneous Cournot, with marginal costs ranging from c to 0. If investments fall in zones III or IV, then firms will be asymmetric after investment, in that the firm with higher R&D investment will be more efficient and therefore will endogenously emerge as a Stackelberg leader; both firms end up with positive marginal costs. In zones

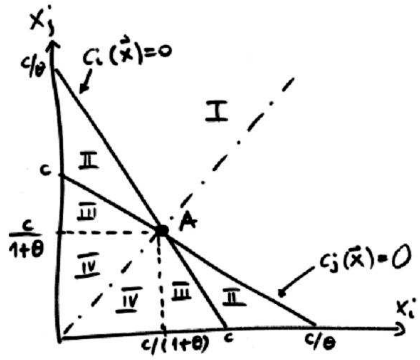

Figure 1: R&D Strategy space

II, there will be also a more efficient firm, but that firm will have zero marginal costs. Finally, in zone I both firms reach the minimum marginal cost possible, i.e. zero. Note that in zone I, despite the fact that R&D investments may be different, firms end up competing in Cournot fashion since they are both equally efficient.

Figure 1 signals one of the complexities of solving for equilibrium, but also why the model is interesting: payoff functions are not continuous and therefore, small changes in a firm's R&D investment may have large impacts, *ceteris paribus*. To show this graphically, in Figure 2 we have graphed the payoff function of one firm as a function of its own investment, keeping constant (at a positive level) the investment of the competitor.[2]

A simple analysis of Figure 2 helps to understand most of what will be happening in the next Sections. When firm i invests little (less than j), between 0 and point *P1* in Figure 2, it ends up being a Stackelberg follower. But surpassing what j is investing, even if marginally, implies a large increase in its payoff because the increase in investment costs is small, but the mode of competition has changed and i is now a Stackelberg leader. This is what we call the *hyper-strategic* effect of R&D investments. Further investments above *P1* would only lower the firms marginal costs, but without changing the mode of competition. But if firm i invests above *P2*, then it will reach zero marginal costs and further investments will decrease only the rival's marginal cost through spillovers, inducing a stronger competitor; this is reflected in a downward-sloping payoff. If investment go further, then there is an extra drop in the payoff function as the rival would also achieve zero marginal cost

[2]The values of the parameters in Figure 2 are: $a = 15$, $c = 1$, $v = 10$, $\theta = 0.5$, $x_j = 0.4$.

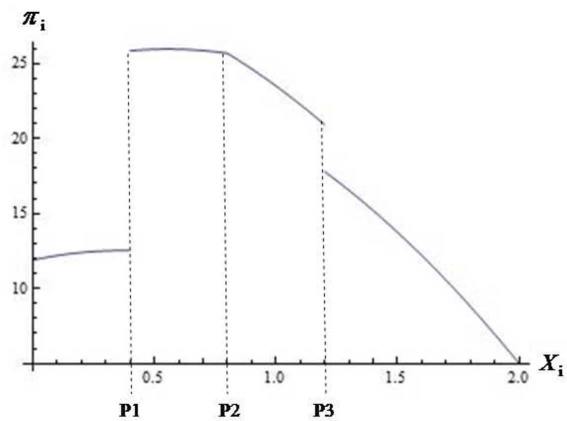

Figure 2: Payoff as a function of own investment

and competition cease to be Stackelberg to become a simultaneous Cournot.

It is important to note, to close this section, that most of the previous literature on R&D focused only on what we call here zones IV and, most specifically, the diagonal *excluding* point A, that is, *interior equilibria*. As we will show, when the timing of the market competition is endogenous, point A plays a major role, which makes a necessity to study the simultaneous market game and the first best for the whole strategy space.

3. Equilibrium with endogenous timing

In this Section, we will search for equilibria when the market competition mode emerges endogenously as in van Damme and Hurkens (1999). We will consider all possible points in the strategy space and, through analysis of unilateral deviations, show that for most relevant cases a sub-game perfect Nash equilibrium (SPNE) is sustained only by investments that leave both firms with zero marginal costs (point A in Figure 1); in other words, there is full R&D investment. Let us start then by considering zone I. If investments are in this zone, any of the two firms can decrease its R&D expenses without affecting its marginal costs because they are already at their minimum attainable; therefore no point in zone I can sustain a SPNE. If we consider zones II, the leader, i.e. the firm with the highest expenditure in R&D, has an incentive to decrease its R&D expenses because, on one hand, it will not affect its own marginal costs –which are already zero– and, on the other hand, it will increase its opponent's marginal cost through diminished spillovers. Thus zones II cannot sustain a SPNE either.

We now jump to the analysis of zones IV, but excluding the frontiers with zones III and point A. Consider first the diagonal, where $x_i = x_j \leq \frac{c}{1+\theta}$, i.e. both firms have positive marginal costs. In this case, each firm has an incentive to increase its R&D investment marginally, thus becoming more efficient and therefore a Stackelberg leader. The intuition is given by Figure 2: on point P1 –where market competition will be Cournot– a marginal increase in R&D expenses generates an incremental change in payoffs. And this will be true as long as there is a space for a firm to *outinvest* the other, i.e. on any point along the diagonal with the exception of point A. We can thus conclude the following:

Proposition 1. *There exists no symmetric pure strategy equilibrium in the R&D investment game that induces positive marginal costs in the subgame.*

The next obvious question is whether asymmetric points in zones IV can sustain a SPNE, since in those cases the follower firm may find it too expensive to outinvest the leader; in other words, the prize may not be large enough. Straightforward use of equations (2) and (3) show that from any point of the type $0 < x_F < x_L < \frac{c}{1+\theta}$ it will always be more profitable for the firm that has less R&D investment to invest marginally above what the competitor did. In other words, this does not sustain a SPNE, as the follower has a profitable deviation to $x_L + \varepsilon$. However, one can show that for a very small interval of values of v and a very small market size –as measured by the ratio a/c– there might be a SPNE with $x_F = 0$ and $0 < x_L < \frac{c}{1+\theta}$. The parameter space where this can happen is very small though, as shown in Figure 3.

We can now turn to the analysis of point $A(c, \theta) = (c/(1+\theta), c/(1+\theta))$, that is, symmetric full investment in R&D. If firms are at this point, the incentive to outinvest the competitor is gone: marginal costs cannot be decreased further and therefore, an extra investment will only increase costs without changing marginal costs nor –more importantly– the mode of competition. Thus, if we are to find incentives for a firm to deviate, it has to be that a firm decides to invest less, decreasing its costs, but accepting to become a Stackelberg follower in the continuation game. It seem quite obvious that this should depend on the relative size of investment costs; intuition dictates that if a firm finds it profitable to deviate, it is because investment costs are large. This intuition in fact holds: straightforward (yet cumbersome) algebra allow us to show that if investments costs v are not *too*

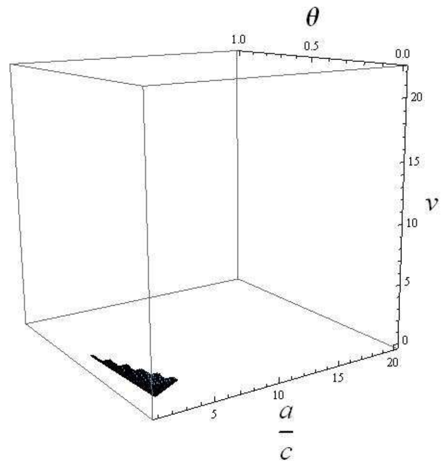

Figure 3: Parameter space where $x_F = 0$ and $0 < x_L < c/(1+\theta)$sustain a SPNE

large, then point A is indeed an equilibrium. The exact thresholds depend on the size of the market as measured by the ratio a/c. Specifically:

- If $\frac{a}{c} \geq \frac{3-2\theta}{1+\theta}$ and $v \leq v_{ET}(\frac{a}{c}, \theta)$ then $A(c,\theta)$ is an equilibrium of the reduced game and therefore sustains a SPNE.
  If $v > v_{ET}(\frac{a}{c}, \theta)$ then it is profitable for a firm to deviate to a smaller yet positive R&D expenditure. The value of v_{ET} is given by:

$$v_{ET}\left(\frac{a}{c}, \theta\right) = \frac{1}{144}\left(7\left(\frac{a}{c}\right)^2(1+\theta)^2 + 18\frac{a}{c}\left(3+\theta-2\theta^2\right)\right) + \frac{1}{144}\left((1+\theta)\sqrt{7}\sqrt{7\frac{a}{c}^2(1+\theta)^2 - 36\frac{a}{c}\left(2\theta^2-3-\theta\right) - 36\left(3-2\theta\right)^2}\right)$$

- If $1 < \frac{a}{c} < \frac{3-2\theta}{1+\theta}$, and therefore $\theta < 2/3$, and if $v \leq v_1(\frac{a}{c}, \theta)$ then $A(c,\theta)$ is an equilibrium of the reduced game and therefore sustains a SPNE.
  If $v > v_1(\frac{a}{c}, \theta)$ then it is profitable for a firm to deviate to zero R&D investment.
  The value of v_1 is given by:

$$v_1\left(\frac{a}{c}, \theta\right) = \frac{1}{72}\left(-9\left(3-2\theta\right)^2 + 7\left(\frac{a}{c}\right)^2(1+\theta)^2 + 18\frac{a}{c}\left(3+\theta-2\theta^2\right)\right)$$

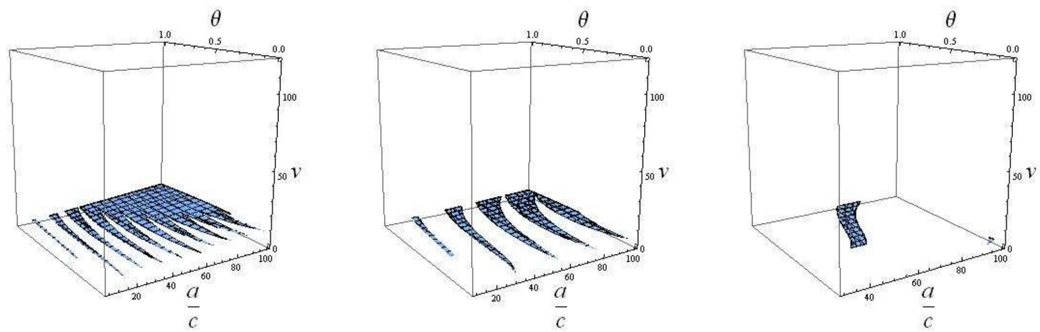

Figure 4: Parameter space areas where there are interior equilibria in zones III (c is 5, 10 and 30)

We can finally analyze what happens in zones III, which can be fairly small when θ approaches 1. What is interesting about these areas is that the firm with smallest R&D investment cannot profitably outinvest the efficient firm this time: any increase in the R&D expenses would first put the competitor's marginal cost to zero, as can be seen in Figure 1. Thus, one can think that it may be the case that *interior* asymmetric equilibria may arise. It is a matter of algebra to show that it is indeed possible that this happens; the parameter space where this might happen, however, is very small, and it shrinks as c grows, as can be seen in Figure 4. Importantly, the interior equilibria in zones III may co-exist together with point A being an equilibrium of the R&D game.[3]

Overall, then, where do we stand with the equilibria analysis? Figure 5 gives us an answer. It shows, in the parameter space, all the values of $v(a/c, \theta)$ that are *small* enough to sustain $A(c, \theta)$ as a SPNE outcome in investment. As can be seen, it is a vast majority of the parameter space and, because of this, we will focus on this case for comparisons to the (exogenously) simultaneous market competition game, and the first best. For values that are above the frontier, i.e. to the left of the steep *wall*, there is no equilibrium in pure strategy R&D investments. There will be however, mixed-strategy equilibria (Simon and Zame, 1990).

One way of thinking about this results, is by a tatonemment process. Suppose that, initially, both firms have agreed to small R&D investments. The hyper-strategic effect of changing the MODE of competition however gives incentives to both to invest just marginally more in order to reap the benefits of market leadership. Thus, pretty much

[3]For example if $a = \frac{385}{128}$, $c = 1$, $v = \frac{251}{128}$, $\theta = \frac{3}{128}$, there are three equilibria: $A(c, \theta)$ and one in each zone III.

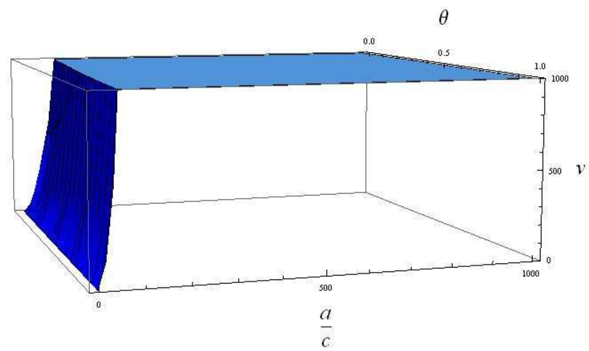

Figure 5: Area where $A(c, \theta)$ is equilibrium (to the right of the frontier)

as in the well-known Bertrand process, firms will compete for having the larger investment until the point at which the prize is no longer large, i.e. point A. Once that point is reached, in some rare cases, one firm may find optimal to go back to smaller investments, saving on costs but accepting to become the follower. In the majority of the cases though, the raise for the market leadership finishes with both firms investing fully. Two final things are worth noticing here. First, the frontier shown in Figure 5 depends on the reduced profit function for the case when both firms end up with identical marginal cost. As explained, we made here the assumption that this payoff is Cournot –something rather in line with the usual oligopoly theory– but there is not certainty yet that this is the case. A different payoff function will change v_{ET} leading to a different frontier. Second, the role of spillovers has been widely studied in the literature. Figure 5 shows, however, that the role of θ is less important in selecting the equilibrium than that of the size of the market or investment costs.

4. Simultaneous subgame and first best

Solving the reduced R&D game when the market competition subgame is exogenously simultaneous (Cournot) is simpler. First, the arguments for zones I and II arguments remain valid. Second, in zones III and IV, payoffs come now from that of simultaneous subgame equilibrium, i.e. that given by equation (4). Now, Brander and Spencer (1983), D'Aspremont and Jacquemin (1988) and Qiu (1997) studied interior equilibria only, by making assumptions on the values of v and a/c. In our case however, given the results obtained for the endogenous timing case, we will need to consider all points in the R&D strategy space, particularly $A(c, \theta)$. Thus, we make no *a priori* assumption regarding v or a/c and solve the fixed point from the best reply functions. We get:

$$x_i^* \equiv x_j^* \equiv x_S \equiv \frac{2\,(a-c)\,(2-\theta)}{9v-4-2\,(1-\theta)\,\theta} \tag{5}$$

Now, this equilibrium is valid only when marginal costs in the subgame are positive. As can be seen, the value of the R&D investments is small when investments costs v are large, and investments are large when the associated costs are small, something rather intuitive. Therefore, for large values of v the equilibrium will be interior and given by equation (5). But what happens when R&D costs are really low? intuition would dictate that in that case, R&D investments would reach the maximum, putting marginal costs in the subgame at its minimum. This is indeed the case if the market is *large enough*.

- Suppose $\frac{a}{c} > \frac{2-\theta}{1+\theta}$. Then,
  - if $v > v_S(\frac{a}{c}, \theta)$ then the unique equilibrium of the game is (x_S, x_S), with $0 < x_S < \frac{c}{1+\theta}$.
  - If, on the other hand, $v \leq v_S(\frac{a}{c}, \theta)$, then the unique equilibrium of the reduced R&D game is $A(c, \theta)$, leading to zero marginal costs.

The value of v_S is given by:

$$v_S\left(\frac{a}{c}, \theta\right) = \frac{a}{c}\,\frac{2a(2-\theta)(1+\theta)}{9}$$

The two regions of the parameter space are shown in Figure 6. In the upper region the equilibrium is interior, leading to positive marginal costs in the subgame; in the lower region the equilibrium is $A(c, \theta)$, i.e. full investment, leading to zero marginal cost.

Note that, again, it seems that the size of the spillover has a mild effect on determining which equilibria will arise. More important are the size of the market and R&D investments costs. When the market is small, and therefore spillovers have to be less than one-half necessarily, the same results hold qualitatively for the simultaneous subgame case, with the exception that the v–thresholds are different, and there is a small region where only mixed strategy equilibria exists.

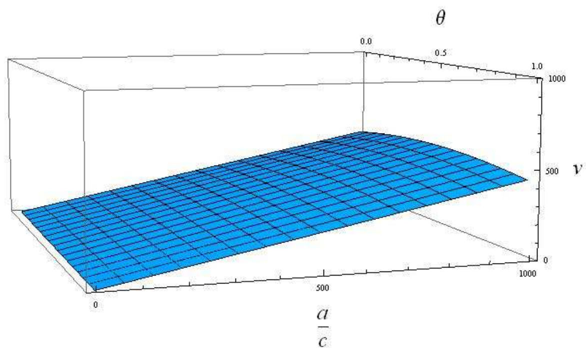

Figure 6: Regions for different equilibrium in the simultaneous case

The comparison of the two cases, when the market is *large enough* can be summarized in the following Proposition:

Proposition 2. *If $\frac{a}{c} > \frac{3-2\theta}{1+\theta}$ $\left(> \frac{2-\theta}{1+\theta}\right)$ then $v_S < v_{ET}$. Consequently, the parameter space for which $A(c,\theta)$ is sustained as equilibrium with simultaneous subgame is strictly contained in the parameter space for which $A(c,\theta)$ is sustained as equilibrium with endogenous timing subgame.*

- *If $v > v_{ET}$, $A(c,\theta)$ is not sustained as equilibrium with endogenous timing subgame; there exists equilibrium in mixed strategies. The equilibrium investment in the game with simultaneous subgame is interior.*
- *If $v_S < v \leq v_{ET}$, $A(c,\theta)$ is sustained as equilibrium with endogenous timing subgame.The equilibrium investment in the game with simultaneous subgame is interior.*
- *If $v < v_S$, $A(c,\theta)$ is sustained as equilibrium in both games.*

The comparison clearly shows the effect of the endogenous timing assumption: the hyper-strategic effect of changing the mode of competition pushes R&D investment above what the strategic effect only predicts.

Next, to be able to perform a complete set of comparisons, we finally need to look for the set of first-best investments and quantities. The social welfare function to be maximized is given by:

$$
\begin{aligned}
SW(x_i, x_j, q_i, q_j) = CS(q_i, q_j) + q_i(a - c - q_i - q_i + x_i + \theta x_j) \\
+ q_j((a - c - q_i - q_i + x_i + \theta x_j) - \frac{v}{2}(x_i^2 + x_j^2) \qquad (6)
\end{aligned}
$$

Previously in the literature (Brander and Spencer, 1983; D'Aspremont and Jacquemin, 1988; Qiu, 1997), the first best was analyzed using the first-order conditions from (6), obtaining a symmetric set of values. However, an analysis of second order conditions show that this approach does not lead to the first-best necessarily: it may be the case that a corner solution is better. For example, if $0 \leq \theta \leq 0.414$ and $v > \frac{a}{c}$, then is socially better a unique firm investing and producing, than both firms following the first-order conditions. An example of this situation can be obtained simply by plugging the values $a = 3$, $c = 1$, $v = 5$ and $\theta = 0$, which lead to a value of SW of 2.2 under first-order condition values, while $SW(0.5, 0, 2.5, 0) = 2.5$. However, in order to increase the comparability of results, we will indeed restrict our attention to symmetric vectors, that is, we will solve the problem

$$\max_{x_O, q_O} CS(q_O, q_O) + 2q_O(a - c - 2q_O + x_O(1+\theta)) - vx_O^2$$

The solution of this problem is simple, and has the same qualitative characteristics of the case with simultaneous subgame, but without a specific requirement about the size of the market:

- If $v > v_O(\frac{a}{c}, \theta)$ then the symmetric first-best is $0 < x_O = \frac{(a-c)(1-t)}{2v-(1+t)^2} < \frac{c}{1+\theta}$ and $q_O = \frac{v(a-c)}{2v-(1+t)^2}$.
- If, on the other hand, $v \leq v_O(\frac{a}{c}, \theta)$ then the symmetric first-best is $A(c, \theta)$ and $q_O = \frac{a}{2}$.

The value of v_O is given by

$$v_O(\frac{a}{c}, \theta) = \frac{a}{c} \frac{(1+\theta)^2}{2}.$$

Finally, it is important to note that, since marginal costs are constant there are constant returns to scale throughout and therefore, R&D investment costs are not recouped in the first-best: firms actually loose money.

5. Comparisons

The comparison of R&D investment levels in the three cases, namely, endogenous subgame, simultaneous subgame and (symmetric) first-best is captured in the following Proposition:

Proposition 3 (Comparison of R&D investments.). *If $\frac{a}{c} > \frac{3-2\theta}{1+\theta}$ then there exist $v_S < v_O < v_{ET}$ such that:*

- *If $v > v_{ET}$, then in the endogenous timing subgame case there is only mixed strategy equilibria in R&D. The other two cases fulfill $x_S < x_O$.*
- *If $v_O < v \leq v_{ET}$, then $x_S < x_O < x_{ET} = A(c, \theta)$.*
- *If $v_S < v \leq v_O$, then $x_S < x_O = x_{ET} = A(c, \theta)$.*
- *If $v < v_S$, then $x_S = x_O = x_{ET} = A(c, \theta)$.*

Figure 7 helps to understand better: the parameter-space can be divided in three regions (leaving aside the one that is at the far left). In the upper region, one finds that in the endogenous sub-game case, R&D investment is at its maximum, above what would arise if the subgame was simultaneous and even above what is first-best. This is what we refer to as the hyper-strategic effect. In the middle region the game with endogenous subgame continues to generate full investment in R&D but, this time, this coincides with the first best, while the simultaneous sub-game keeps generating lower levels of R&D. Note here however, that the full game under endogenous timing subgame cannot be the actual first-best since there will be a market power effect in the market competition level. Finally, in the lower zone, when investments costs are very small, both the endogenous timing subgame and the simultaneous subgame produce full R&D investment, something that coincides with the symmetric first-best. Something noticeable is that in the case of the first-best, the size of the spillover does matter, much more than in the oligopoly cases.

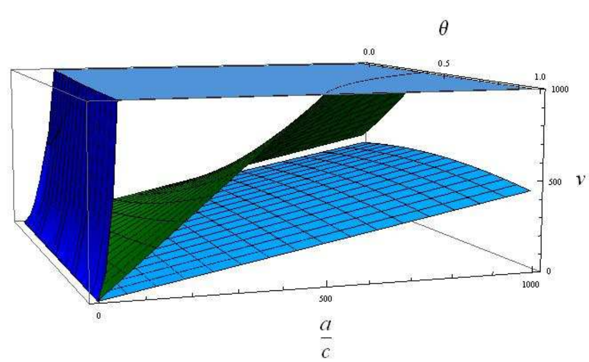

Figure 7: Parameter space divided according to R&D investments

The next step in these comparisons, is to study what happens with production at the market competition stage. Note first that the endogenous-timing nature of the subgame had a strong influence in the result of the R&D game yet, at the end, both firms end up investing the same and therefore the quantity game is indeed a simultaneous Cournot. Next, the comparison between the simultaneous case and the endogenous-timing case is simple: since under Cournot competition production levels are a decreasing function of marginal cost, and these are in turn decreasing functions of R&D investments, production will be weakly smaller in the case of the simultaneous subgame. The same reasoning, plus the fact that Cournot production levels are always smaller than perfect competition production levels, allow us to conclude that q_O will be strongly larger than q_S. The comparison between the first-best quantities and the endogenous-timing quantities however is, in principle, a little bit more difficult. In the middle and lower parts of Figure 7 both the first best and the endogenous-timing subgame lead to zero marginal cost and, therefore, q_O will be larger than q_{ET} since in the latter there is market power. But what happens in the upper region, where the endogenous timing subgame induces larger R&D investments than the first best, and therefore, smaller marginal costs? Well, it happens that it is easy to argue that the market power effect dominates the gains in efficiency. On one hand, by construction it is true that $SW_O > SW_{ET}$; but also, it is true that $\pi_O < 0 < \pi_{ET}$. Thus, given the definition of SW it must be the case that $CS_O > CS_{ET}$, and since for symmetric production levels $CS = q^2/2$, one finally obtains that $q_O > q_{ET}$. We summarize these analysis in the following Proposition.

Proposition 4 (Comparison of sub-game quantity levels.). *If $\frac{a}{c} > \frac{3-2\theta}{1+\theta}$ then there exist $v_S < v_O < v_{ET}$ such that:*

- *If $v_S < v \leq v_{ET}$, then $q_S < q_{ET} < q_O$.*
- *If $v < v_S$, then $q_S = q_{ET} < q_O$.*

The final relevant comparison has to do with social welfare levels. Clearly $SW_S < SW_O$ and $SW_{ET} < SW_O$, but how do the two oligopoly cases compare?. Of course, this comparison is relevant in the region of the parameter space in which the R&D levels differ, i.e., when the simultaneous subgame case delivers interior R&D equilibrium. In those cases, the ET game delivers smaller marginal costs and therefore larger quantities, which benefit consumers. We can then conclude that $CS_S < CS_{ET}$. However the smaller marginal costs

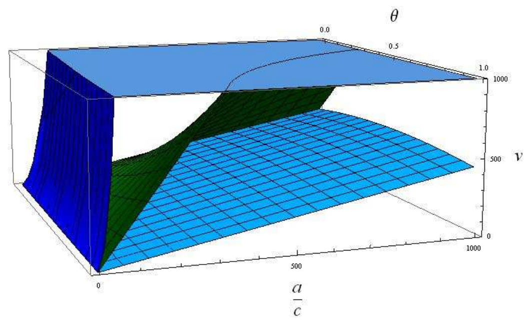

Figure 8: SW comparisons between Endogenous-timing and simultaneous cases

come from investments on R&D which are costly, and therefore profits comparisons are not straightforward, making welfare comparisons uncertain. In fact, it is the case that for some parameter values, consumers benefit more than what the firms loose from the extreme R&D competition, inducing a superior results in terms of welfare for the endogenous-timing case; while for other parameter values, there is simply too much investment in expensive R&D and resources are wasted, despite the fact that consumers enjoy higher production levels and lower prices. In Figure8 we show the regions of the parameter space where each of these situations take place: In the upper region $SW_S > SW_{ET}$ while in the middle region $SW_S < SW_{ET}$. But, as explained, in the middle upper region it happens that $\pi_S > \pi_{ET}$, while in the middle lower region it happens that $\pi_S < \pi_{ET}$.

6. Conclusions

If firms know that market leadership depends on cost structures, the effect that process R&D may have on market structure may turn firms into be more aggressive when seeking for cheaper technologies. To further explore the scope of the advantage that process R&D may give to competing firms we have presented a two stage R&D model with endogenous timing in which after simultaneous investment in R&D, the most efficient firm acts as a Stackelberg leader in the production stage.

Our findings are that in this environment there is no symmetric equilibrium with positive marginal costs after investment. If investment costs are *not extremely large*, there exists a full investment symmetric (sub game perfect) equilibrium in which firms reduce there costs to their lower bounds. This is caused by the *hyper-strategic* effect that now R&D investment has, as it does not only increase efficiency but changes the mode of competition by creating market leadership. The value of equilibrium marginal costs does not depend on the size of spillovers.

This equilibrium outcome differs radically from previous works. We show that in the vast majority of the parameter space, firms invest more in R&D than when the market competition is exogenously simultaneous and, in fact, R&D investments are weekly larger than the First-Best ones.

If investment costs are large, firms overinvest in endogenous timing with respect to social optimum while if investment costs are small, investment with endogenous timing is equal to social optimum.

Regarding the case of simultaneous subgame. We find that welfare comparison between this situation and endogenous timing subgame, depends strongly on firms profits. These larger R&D investments, that lead to decreased costs of production, while beneficial for the consumers, may in some cases hurt the firms enough to actually diminish social welfare as compared to the simultaneous market competition case. However, if investment costs are not too large and spillovers are big, both firms and consumers are better of with endogenous timing than sequential subgame.

REFERENCES

Amir, R., I. Evstigneev, and J. Wooders (2003). Noncooperative versus cooperative R&D with endogenous spillover rates. *Games and Economic Behavior 42*(2), 183 – 207.

Amir, R. and I. Grilo (1999, January). Stackelberg versus cournot equilibrium. *Games and Economic Behavior 26*(1), 1–21.

Amir, R. and A. Stepanova (2006). Second-mover advantage and price leadership in bertrand duopoly. *Games and Economic Behavior 55*(1), 1–20.

Amir, R. and J. Wooders (2000). One-way spillovers, endogenous innovator/imitator roles, and research joint ventures. *Games and Economic Behavior 31*(1), 1 – 25.

Atallah, G. (2005). R&D cooperation with asymmetric spillovers. *The Canadian Journal of Economics / Revue canadienne d'Economique 38*(3), 919–936.

Brander, J. A. and B. J. Spencer (1983). Strategic commitment with R&D: The symmetric case. *The Bell Journal of Economics 14*(1), 225–235.

D'Aspremont, C. and A. Jacquemin (1988). Cooperative and noncooperative r & d in duopoly with spillovers. *The American Economic Review 78*(5), 1133–1137.

Fudenberg, Drew Tirole, J. (1984). The fat-cat effect, the puppy-dog ploy, and the lean and hungry look. *The American Economic Review 74*(2), 361–366.

Hamilton, J. H. and S. M. Slutsky (1990). Endogenous timing in duopoly games: Stackelberg or cournot equilibria. *Games and Economic Behavior 2*(1), 29–46.

Harsanyi, J. and R. Selten. (1988). *A General Theory of Equilibrium Selection in Games*. Cambridge, MA: MIT Press.

Kambhu, J. (1984). Uncertainty and endogenous stackelberg equilibrium. Mimeo, Department of Economics, Columbia University.

Mailath, G. J. (1993). Endogenous sequencing of firm decisions. *Journal of Economic Theory 59*(1), 169–182.

Qiu, L. D. (1997). On the dynamic efficiency of bertrand and cournot equilibria. *Journal of Economic Theory 75*(1), 213–229.

Sadanand, A. and V. Sadanand (1996). Firm scale and the endogenous timing of entry: a choice between commitment and flexibility. *Journal of Economic Theory 70*(2), 516–530.

Sadanand, V. and E. Green (1991). Firm scale and the endogenous timing of entry: A choice between commitment and flexibility. Working papers, University of Guelph, Department of Economics.

Saloner, G. (1987). Cournot duopoly with two production periods. *Journal of Economic Theory 42*, 183–187.

Simon, L. K. and W. R. Zame (1990). Discontinuous games and endogenous sharing rules. *Econometrica 58*(4), 861–872.

Spencer, B. J. and J. A. Brander (1992). Pre-commitment and flexibility : Applications to oligopoly theory. *European Economic Review 36*(8), 1601–1626.

Suzumura, K. (1992). Cooperative and noncooperative R&D in an oligopoly with spillovers. *The American Economic Review 82*(5), 1307–1320.

van Damme, E. and S. Hurkens (1999). Endogenous stackelberg leadership. *Games and Economic Behavior 28*(1), 105–129.

van Damme, E. and S. Hurkens (2004). Endogenous price leadership. *Games and Economic Behavior 47*, 404–420.