Alameda 3363
Estación Central-Santiago
Tel. +56 2 7180765
http://www.fae.usach.cl/economia/

Universidad de Santiago

Departamento de Economía

Serie de Documentos de Trabajo

Precautionary Savings in General Equilibrium Consumption

Autores:
Facundo Sepúlveda (Universidad de Santiago)

WP 2014 - Nº 02

Precautionary Savings in General Equilibrium

Facundo Sepulveda [1]
Universidad de Santiago de Chile

January 2013

[1] Address: Economía USACH. Alameda 3363. Estación Central, Santiago, Chile. E-mail:facundo.sepulveda@fsp.cl

Abstract

This paper uses a calibrated stochastic OLG model to address three questions about US savings and wealth accumulation: first, does an equilibrium display buffer stock savings by agents? Second, is this equilibrium consistent with savings behavior of US households? And finally, what level of precautionary savings arises when general equilibrium effects are accounted for? We find that given observed earnings risk, the rates of time preference that are consistent with the equilibrium are very close to the interest rate, so no buffer stock behavior is observed. Moreover, the equilibrium reproduces important facts about savings behavior of US households. Finally, accounting for general equilibrium effects lowers the size of precautionary wealth to about 35% of aggregate wealth, or 30 to 50% less than partial equilibrium estimates.

1 Introduction

It is now understood that precautionary motives for accumulating wealth play a key role in the consumption/savings decisions of households. At least since the work of Skinner [1988], Hubbard and Judd [1987], and Summers and Carroll [1987], precautionary savings behavior has been extensively studied, primarily as a candidate solution to problems in the consumption literature, such as the excess sensitivity puzzle (Zeldes [1989], Caballero [1991]), and the failure of standard finite horizon models to explain the observed pattern of consumption growth over the life cycle (Skinner [1988]). Alternatively, precautionary motives have been advanced to link the decline in the personal savings rate over the last 20 years to the extension of social insurance programs such as Medicare and Social Security (Summers and Carroll [1987]).

This paper is concerned with a research agenda fostered by Skinner [1988], Carroll and Samwick [1998, 1997], Hubbard et al. [1994] (HSZ), Huggett [1996] and others. The objective is to study whether a model with realistic lifespans, income paths, and risk exposure can account for the savings/consumption behavior of US households. In this line of work, Hubbard et al. [1994] showed that in a calibrated model where the interest rate is close to the rate of time preference, agents would desire to accumulate levels of wealth similar to those found in the data. Moreover, evidence was reported that other model statistics such as the age-consumption profile, and the response of consumption to innovations in income could also reproduce their data counterparts. In a companion paper (Hubbard et al. [1995]) these authors focus on the importance of asset tested programs to explain the low accumulation of assets by the lowest quintile of the wealth distribution.

The calibration of these models was criticized by Carroll and Samwick (Carroll and Samwick [1998, 1997]), on the grounds that it produces a level of sensitivity to changes in income risk so high that it was impossible to reconcile with their empirical findings. Instead, they propose a calibration where agents have very high levels of impatience, so that the rate of time preference is well above of the interest rate. In such model, agents find it optimal to achieve a target level of wealth over (expected) income, which they keep until late in their life cycle. Carroll and Samwick report that this model displays a sensitivity to changes in income risk more in line with their empirical results.

One common finding of this literature is that wealth that is held for precautionary motives accounts for at least 50% of total wealth. However, these estimates are partial equilibrium in nature, as prices do not respond to changes in aggregate wealth [1]. As shown by Hubbard and Judd [1987] in a model with longevity risk only, and by Aiyagari [1994] in the context of an infinite horizon model, general equilibrium effects can be sizable and tend to increase wealth holding, therefore reducing the estimated share of wealth that is precautionary.

[1] An exception is Huggett [1996] who carries a general equilibrium analysis and reports a lower estimate. However, his focus is on wealth distribution, so there is no discussion of this result.

This paper contributes to the literature by taking an alternative path: imposing the discipline of general equilibrium, we compute the levels of discount rates consistent with observed levels of interest rates, savings rates, and income/longevity risk. We show that the resulting equilibrium produces interest and discount rates that are very close to each other, so that agents are not buffer stock savers. Moreover, the age specific saving behavior that emerges is consistent with average asset accumulation by US households, and displays levels of sensitivity to income risk in line with those reported by Carroll and Samwick. Finally, we compute the level of precautionary savings that arise in this model, and show that properly accounting for general equilibrium effects considerably lowers previous estimates.

This paper has four other sections. In section 2 the model is presented. Section 3 discusses the calibration procedure. Section 4 presents the results, and Section 5 concludes.

2 The model

We present a large scale OLG model in the tradition of İmrohoroğlu et al. [1995], Huggett [1996] and Rios-Rull [1996]. In this model, a large number of agents of size 1 live for a maximum of T periods, are endowed with a level of assets a_1 at the beginning of their life ($t = 1$) and face uncertainty regarding labor earnings and lifespan. Each period, agents take the interest rate and the realization for labor income as given and must allocate their earnings between consumption and saving, subject to a borrowing constraint.

Agents take prices as given and maximize a utility function of the form:

$$\max_{c_t, a_{t+1}} E_1 \sum_{t=1}^{T} [\prod_{j=1}^{t} \eta_j] \beta^t u(c_t)$$

$$\begin{aligned} s.t. \quad a_{t+1} + c_t &= (1+r)a_t + w\phi_t l_t + q \\ a_{t+1} &\geq 0, \end{aligned}$$

where l_t, is a random variable with bounded support that represents a shock to labor endowment, and ϕ_t is a nonstochastic variable that indexes labor productivity for an agent of age t. Therefore, $w\phi_t$, is the unconditional mean of labor earnings at age t, that can be thought of as the life cycle component of earnings, and l_t, is labor endowment of an agent at age t.

An agent of age t survives to $t+1$ with probability η_t. With probability $1-\eta_t$, he dies and leaves bequests that are evenly distributed among all living agents, each agent receives q in bequests every period. Survival probabilities $\{\eta_t\}_{t=1}^{T}$, in turn define the cohort shares $\{\mu_t\}_{t=1}^{T}$ by $\mu_t = (1-\eta_t)\mu_{t-1}$ and $\sum_{t=1}^{T} \mu_t = 1$.

The household problem can be expressed in recursive form. Let $V_t(a, l)$ be the maximum value of the objective function for an agent of age t with a level of asset holdings and labor endowment shock $\{a, l\}$. Then, $V_t(a, l)$ is given by:

$$\begin{aligned} V_t(a, l) &= \max_{a', c} \{u(c) + \beta \eta_t E[V_{t+1}(a', l'|l)]\} \\ s.t. \quad a' + c &= (1+r)a + w\phi_t l + q \end{aligned}$$

$$a' \geq 0 \qquad (P1),$$

where a' is asset holdings for next period. Moreover, since an agent lives at most for T periods, we have:

$$\begin{aligned} V_T(a,l) &= \max_c \{u(c)\} \\ s.t. \qquad c &= (1+r)a + w\phi_t l + q \end{aligned}$$

The solution to this problem are the optimal policy functions $C_t(a,l)$ and $A_t(a,l)$, for $t = 1, ..., T$, that map the state $\{a, l\}$ at age t to consumption at age t and assets at the beginning of age $t+1$ respectively.

The representative firm chooses $\{L, K\}$ to solve:

$$\max_{K,L} F(K,L) - RK - wL \qquad (P2).$$

To complete the description of the economy, we define the capital accumulation technology in a standard way: $K_{t+1} = (1-\delta)K_t + I_t$, where I is aggregate investment and δ is the depreciation rate.

We are interested in a steady state equilibrium where the aggregate capital stock is constant, and although there is a large amount of dynamics at the individual level, the distribution of assets and other endogenous variables is time invariant. Since a meaningful equilibrium concept needs to be expressed in terms of these distributions, we proceed to define them.

Let $(X, \mathcal{B}, \Psi_t)$ be a probability space. If Z is the support for the stochastic shock l_t and asset holdings are restricted to lie in $[0, \infty)$, then an individual state $x = \{a, l\}$ lies in the state space $X = Z \times [0, \infty)$. Let B be the Borel sets in X. Then, for each t from 1 to T a distribution Ψ_t can be defined such that, for each $B \in \mathcal{B}, \Psi_t(B)$ is the probability that an agent of age t will be in a state $x \in B$. Together with the stochastic process for l, the optimal policy function $A_t(a, l)$ defines a transition function $P(B, t) = Prob(x_{t+1} \in B|x_t)$ that links current and future distributions. The function Ψ_t is then derived recursively by:

$$\Psi_t(B) = \int_X P(B, t-1) d\Psi_{t-1} \quad B \in \mathcal{B}.$$

Equilibrium Definition: A steady state equilibrium for this economy is a collection of value functions $V_t(.)$, policy functions $C_t(.)$ and $A_t(.)$, $t = 1, ..., T$; prices for labor and capital services $\{w, r\}$; aggregate values for $\{K, L\}$; a level of per capita bequests q; and distributions $\{\Psi_t, P_t\}$ for $t = 1, , T$ such that,

1. Households maximize utility: given q and prices $\{w, r\}$, the policy functions $C_t(.)$ and $A_t(.)$ solve $(P1)$ for all t.

2. Firms maximize profits:

   $F_K = R = r + \delta$

   $F_L = w.$

3. Markets clear:

(i) $\sum_t \mu_t \int (C_t + A_t) d\Psi_t + q = (1-\delta)K + F(K, L)$

(ii) $\sum_t \mu_t \phi_t = L = 1$

(iii) $\sum_t \mu_t \int A_t d\Psi_t = K$

4. Cross section distributions are consistent with policy functions:

   $\Psi_{t+1} = \int P_t d\Psi_t$

5. All bequests are distributed:

   $q = \sum_t (1 - \eta_t) \int A_t d\Psi_t$

Aiyagari [1994] presents a characterization of this problem in the context of an infinite horizon model: agents will overaccumulate assets, with respect to a complete markets situation, as a way to partially insure themselves against the possibility of being effectively borrowing constrained in the future.

The pattern of wealth accumulation is studied by Carroll [1999] in a life cycle model, and Deaton [1991] in an infinite horizon economy. An important result is that, when the growth rate of income is sufficiently high for given levels of risk $\{\rho, \sigma_\epsilon^2\}$, prudence $\{\frac{\theta u_c^{'''}}{u_c^{''}}\}$, and patience $\{\beta\}$, agents optimally choose to achieve a target level of wealth over earnings, or "buffer stock" of assets [2].

[2]The condition in discrete time can be approximated by $\frac{r-\gamma}{\theta} + \frac{(\theta+1)\sigma_\epsilon^2}{2} < g$, where g is the growth rate of income, γ is the rate of time preference, and θ is the coefficient of relative risk aversion.

Checking whether this condition is empirically plausible is difficult given the unobservable nature of the discount rate $\gamma = \frac{1-\beta}{\beta}$. The next section presents a calibration procedure where β is determined using the general equilibrium nature of the model.

3 Calibration.

The calibration exercise is designed so that the stochastic model economy displays relevant features of the US economy. In particular, the discount factor β is left as a free parameter that takes on the value needed for the model economy to display target levels of the interest rate and the savings rate.

To calibrate the model we need to define functional forms and parameters. The functional forms used are as follows:

- A Cobb-Douglas production function is used for all exercises:

$$F(K, L) = AK^{\alpha}L^{1-\alpha}$$

  .

- Felicity functions are of the CRRA form:

$$u(c) = \frac{c^{1-\theta} - 1}{1 - \theta}$$

- The stochastic process for the labor endowment is $AR(1)$:

$$\ln(l_t) = \rho \ln(l_{t-1}) + \epsilon_t \qquad \epsilon_t \sim N(0, \sigma_\epsilon^2),\ i.i.d.$$

The parameter values are shown in table 1. For the earnings process we adopt the results for households with 12-15 years of education reported by Hubbard et al. [1994](table A.4) using PSID data. Figure 1 shows the unconditional means. The stochastic process associated implies values of .946 for ρ , and .025 for σ_ϵ^2. These values are roughly consistent with findings by MaCurdy [1982], as explained below, but imply a variance in the change of earnings lower than the values in Abowd and Card [1989], who use the same dataset.

The baseline economy is also calibrated so as to display the following ratios: an interest rate of 4% per annum, in line with the calculations reported by Kotlikoff and Summers [1981], and a savings rate of 19%, chosen to match the rate of investment over GDP of the US economy in the period 1980-1989. These ratios imply a depreciation rate of .045 per year.

A comment of the calibration choices is in order. Evidence from longitudinal studies of earnings and labor supply suggest that the process for (log) earnings can best be modelled as a near unit root process with autoregressive errors of order 2 (MaCurdy [1982]. This leads Carroll and Samwick [1997] to calibrate their model using a unit root process with a variance of innovations equal to 0.01.

As suggested by Skinner [1988], we can summarize the risk to lifetime resources implied by a AR(l) stochastic process with the statistic $\pi_t = \sigma_\epsilon^2[\sum_{j=1}^{T-t} \frac{\phi_{t+j-1}\rho^j}{(1+r)^j}]^2$, where ϕ_t indexes labor earnings at each age. We compute this statistic (the average over all ages) for our baseline parameters, and compare it with those for Carroll and Samwick and MaCurdy, properly accounting for the $ARMA$ specification. We find that the $AR(l)$ process chosen here implies a similar level of risk to lifetime resources (2.54) than the $ARMA(1,2)$ proposed by MaCurdy (2.81) and the specification used by Carroll and Samwick (3.13). Moreover, increasing the variance of innovations from our baseline of .025 to .031 would be enough to produce a value of 3.13 for this statistic.

With respect to the interest and savings rates, since the empirical equivalent of the model interest rate is a risk-free rate, we are tempted to use a number in the order of 1/2% per annum, consistent with the return on Treasury Bonds. On the other hand this rate is also the marginal product of capital, so the historical return on stocks, of the order of 7% annually but more volatile, may also be appropriate. We therefore experiment with different values of r.

The problem of interpreting the savings rate lies in the fact that households are not the only source of savings in the US economy. In fact, from 1980 to 1989 the personal savings rate averaged 6.7% of GDP, businesses contributed with 12.6 percentage points to the average savings rate, and the government dissaved .8% of GDP. Of these aggregates, businesses and government reported as capital consumption allowances 10.2 and 2.3% of GDP respectively. Aggregate gross saving was then in average 18.8% of GDP in this period, close to our benchmark, but the net saving rate was only 5.9% (ERP [1999]).

A related issue is that, since we focus on a steady state equilibrium, savings net of capital consumption (depreciation) allowances is zero in the model. Since introducing growth considerations is beyond the scope of this paper, we check the robustness of our results to the choice of savings rate by doing some sensitivity analysis with values of S/Y from 15% to 24%.

Finally, note that Table1 show the levels of K/Y implied by each choice of $\{r, S/Y\}$. These levels are roughly consistent with the ratios of Assets/Income reported by Hubbard et al. [1994], but are in general higher than the capital-output ratios calculated for the US economy. Note that once the interest rate and savings rates are fixed, the depreciation rate and the capital output ratio are defined by the conditions $s = \delta K/Y$ and $r = F_K - \delta$. Table 7, with the capital output ratios that result from selected $\{r, S/Y\}$ pairs, show that a lower K/Y is associated with higher levels of saving rates and interest rates. While decreasing K/Y to 3 increases the estimated share of wealth that is precautionary (see table 3), it remains that this variation is small compared to the differences in precautionary wealth associated with different risk aversion coefficients.

4 Results.

In this section, we begin by showing how the calibrated model reproduces important facts about wealth accumulation by US households. We then examine the implications of these results for the debate on whether US households are buffer stock savers or not. Finally, we compute the levels of precautionary wealth that emerge in this model and decompose it in partial and general equilibrium effects.

We evaluate the ability of the model to mimic US data along three dimensions: the age/wealth profile, the age-specific average propensities to save, and the sensitivity of wealth holdings with respect to income risk.

The paper focuses on age-specific aggregate statistics, rather than distributions, because we believe that most of the intra cohort heterogeneity cannot be explained by different histories of shocks that are mean-reverting. It is known that this model compresses the income distribution, generating too few very rich (see e.g. Carroll [2000]) and too few very poor agents. Realistic models of wealth distribution imply types of heterogeneity in agents that are absent here: in investment opportunities (for instance Quadrini [2000]), in time discounting (e.g. Krusell and Smith [1998]), or in productivity (e.g. Hubbard et al. [1994]. Considering these types of heterogeneity is beyond the scope of this work.

Figure 2 shows the predicted average profile of wealth holdings at each age (in thousands of 1984 dollars), compared to data reported by Radner [1989] using the 1984 Survey of Income and Program Participation (SIPP) database. The model generated data is normalized so that average income equals 1984 per capita GDP at current prices.

The fit is extremely good given that the model was calibrated to savings and interest rates of the period only. The feature that deserves attention is the similarity of the shapes of the two curves, more than the fact that they overlap. In fact, since SIPP data comprises only private wealth, while the model's data is on aggregate wealth, and their measurement units (households in SIPP versus 'workers' in model) differ, there is little reason why they should overlap. The similarities however suggest that, on average, the model contains the right elements that shape life cycle savings behavior.

Figures3 and 4 present the age-wealth profiles for alternative parameterizations of the model, compared to Radner's data. Clearly, very high levels of aggregate wealth can be attained by this model with the appropriate discount rate.

We now turn to examine two direct measures of age-specific saving behavior. Figure 5 compares the life cycle profiles of average propensities to save (APS) generated by the model with their data counterparts, constructed by Gokhale et al. [1996] using the Consumer Expenditure Survey (CEX) for various years. The two series correspond to different definitions of disposable income. Conventional disposable income is the sum of labor income, capital income, and pension income minus net taxes, while in the alternative definition social security contributions are classified as loans (so that they are considered savings), and social security benefits are classified as the repayment of principal (not part of disposable income) plus interest on past social security loans.

It is important to note that, once again, it is the shape of the APS curve and not the level that matters the most. Data on household savings is data on net savings, since businesses make most of the allowances for consumption of capital. In a growing economy this measure of savings should be positive in the aggregate. In our model, since we are focusing on a Steady State equilibrium, net savings are zero.

The model prediction follows more closely the APS observed under the alternative definition, but it tends to overpredict savings rates at the beginning (until around age 32) of the life cycle. Overall, it displays the characteristic hump shape present in the data, with a 'plateau' from ages 35 to 60, and a drastic decrease after age 60.

Figure 6 allows an examination of the sensitivity of wealth holdings with respect to uncertainty, measured in this case by the conditional variance of earnings σ_{ϵ}^2 . The wealth/income profile for the baseline model is shown along with the average age-wealth profile of an agent facing the same prices as in the baseline model but with half of the variance (1.2% versus 2.4%). The results for two alternative parameterizations are shown in figures 7 and 8.

These simulated changes in the levels of wealth holdings are consistent with those predicted by the regression coefficients in Carroll and Samwick [1997]. Using differences in occupation specific income risk, Carroll and Samwick regress various measures of log net worth on the variance of permanent and transitory income shocks, permanent income, and life-cycle variables (age, married, etc). Using the approximation $[log(Wl)log(W2)]/[var1-var2]$ suggested by the authors, where $W1$,$W2$, and $var1$, $var2$ are wealth holdings and income variances for the baseline and alternative paths respectively, we can approximate what the regression coefficients would be in the models. The results, presented in Table 2, indicate that reasonable parameterizations of the model can reproduce these coefficients without difficulty.

These levels of risk sensitivity are similar to those reported by Carroll and Samwick [1997], even though the levels of discount rates and other parameters are very close to those of Hubbard et al. [1994]. In fact, figures 6, 7, and 8 show that the ratio of wealth/income chosen by agents, increases from the beginning of the life cycle, instead of remaining constant for the first part of it -after a target level is reached- as would be the case in a buffer stock model.

Using the discount factors consistent with Steady State equilibria in the stochastic economies, we can predict how large would aggregate wealth be in a similar economy with no income uncertainty, and no income or lifespan uncertainty. We do so by using a certainty version of the program, described in the appendix.

The results are shown in Table 3. It is worth noting that the levels of precautionary wealth are significantly lower than those found in similar models. Table 4 shows that the difference can be entirely explained by not accounting for general equilibrium effects. In what follows, we examine this issue more closely.

Two exercises are carried out. First, we calibrate our model to interest rates, discount rates, and stochastic paths similar to those in Skinner [1988] and Hubbard et al. [1994]. Rather than attempting a detailed replication, we want to find if given these rough similarities, our model generates similar levels of precautionary wealth. Table 5 shows the results for two different levels of aggregate earnings, and confirms that in partial equilibrium this type of model generates high levels of precautionary wealth.

Next, we compare the levels of precautionary wealth generated by these models in general vs. partial equilibrium (Table 6). Given the parameters for the stochastic process and the interest rate chosen in the original papers, we find the discount factor consistent with a predetermined savings rate (S/Y=.19 for Skinner and .24 for Hubbard et al.). Next, we find the level of aggregate wealth in a deterministic economy where agents face the same factor prices (columns labelled P.E.), and finally we allow prices to change and compute the general equilibrium effects (columns labelled G.E.).

It is clear that the partial vs. general equilibrium nature of the exercise matters, as was already noted in Hubbard and Judd [1987] and Aiyagari [1994]. In our examples, a partial equilibrium estimation of the size of precautionary wealth overstates it by 20 to 50%, consistent with the differences between the findings in this paper and those reported by Skinner [1988] and Hubbard et al. [1994].

5 Conclusions.

An important problem in the study of life cycle savings behavior is whether it can be characterized by a model of buffer stock versus 'life cycle' savings. This paper examines the issue using the discipline of general equilibrium to sort among alternative models.

We find that a model calibrated to the levels of aggregate savings, interest rates, and risk exposure found in the data displays life cycle patterns of asset accumulation and overall sensitivity to risk in line with empirical evidence. This model does not predict buffer stock behavior. Rather, agents find it optimal to increase their wealth/income ratios until shortly before retirement.

At the same time, the equilibrium allocation implied a level of precautionary wealth around 35% of total wealth, far below comparable estimates in the literature. The differences can clearly be traced to the partial/general equilibrium nature of the exercises.

References

Economic Report of the President. United States Government Printing Office, 1999.

Abowd and Card. On the covariance structure of earnings and hours changes. *Econometrica*, 57(2):411–46, 1989.

Rao Aiyagari. Uninsured idiosyncratic risk and aggregate savings. *Quarterly Journal of Economics*, 109:659–84, 1994.

Ricardo Caballero. Earnings uncertainty and aggregate wealth accumulation. *American Economic Review*, 81:859, 1991.

Christopher Carroll. Buffer stock saving: some theory. Manuscript, 1999.

Christopher Carroll. Why the rich save so much? In Joel Slemrod, editor, *Does Atlas Shrug? The Economic Consequences of Taxing the Rich*. Harvard University Press, 2000.

Christopher Carroll and Andrew Samwick. The nature of precautionary wealth. *Journal of Monetary Economics*, 40:41–71, 1997.

Christopher Carroll and Andrew Samwick. How important is precautionary saving? *The Review of Economics and Statistics*, 80(3):410–19, 1998.

Angus Deaton. Saving and liquidity constraints. *Econometrica*, 59:1221–48, 1991.

J. Gokhale, Lawrence Kotlikoff, and J. Sabelhaus. Understanding the post-war decline in us savings: A cohort analysis. *Brooking Papers on Economic Activity*, pages –, 1996.

G. Hubbard and K.L. Judd. Social security and individual welfare: Precautionary saving, liquidity constraints, and the payroll tax. *American Economic Review*, 77:630–46, 1987.

G. Hubbard, J. Skinner, and S. Zeldes. The importance of precautionary motives in explaining individual and aggregate saving. *Carnegie-Rochester Conference Series on Public Policy*, 40:59–125, 1994.

G. Hubbard, J. Skinner, and S. Zeldes. Precautionary savings and social insurance. *Journal of Political Economy*, pages 360–99, 1995.

M. Huggett. Wealth distribution in life-cycle economies. *Journal of Monetary Economics*, 38:469–94, 1996.

Ayşe İmrohoroğlu, Selahattin İmrohoroğlu, and Douglas Joines. A life cycle analysis of social security. *Economic Theory*, 6:83–115, 1995.

Ayşe İmrohoroğlu, Selahattin İmrohoroğlu, and Douglas Joines. Computing models of social security. In Ramon Marimon and Andrew Scott, editors, *Computational Methods for the Study of Dynamic Economies*. Oxford University Press, 1999.

L. Kotlikoff and L. Summers. The role of intergenerational transfers in aggregate capital accumulation. *Journal of Political Economy*, 89(4):706–32, 1981.

Per Krusell and A. Smith. Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy*, 106(5):867–96, 1998.

Thomas MaCurdy. The use of time series processes to model the error structure of earnings in a longitudinal data analysis. *Journal of Econometrics*, 18:83–114, 1982.

Vicenzo Quadrini. Entrepreneurship, savings and social mobility. *Review of Economic Dynamics*, 3, 2000.

Radner. The wealth of the aged and nonaged. In R. Lipsey and S. Tice, editors, *The Measurement of Saving, Investment, and Wealth*. The University of Chicago Press, 1989.

J. Victor Rios-Rull. Computation of equilibria in heterogeneous agent models. In Ramon Marimon and Andrew Scott, editors, *Computational Methods for the Study of Dynamic Economies*. Oxford University Press, 1999.

Jose Victor Rios-Rull. Life-cycle economies and aggregate fluctuations. *Review of Economic Studies*, 63:465–489, 1996.

J. Skinner. Risky income, life cycle consumption, and precautionary saving. *Journal of Monetary Economics*, 22:237–55, 1988.

Lawrence Summers and Christopher Carroll. Why is us national saving so low. *Brooking Papers for Economic Analysis*, 2:607–635, 1987.

G. Tauchen. Finite state markov chain approximations to univariate and vector autoregressions. *Economic Letters*, 20:177–181, 1986.

S. Zeldes. Optimal consumption with stochastic income:Deviations from certainty equivalence. *Quarterly Journal of Economics*, 104(2):275–298, 1989.

A Numerical methods.

To simulate the stochastic OLG economy we use a standard dynamic programming method. To use this method we discretize the state space. In particular, a seven-state discrete approximation to the labor endowment process is used. This approximation is done with the method described in Tauchen [1986] to find a markov transition matrix for continuous stochastic processes.

A.1 Solution Method for the Stochastic OLG Model

The stochastic OLG model is solved using a variation of the İmrohoroğlu et al. [1995] algorithm to compute the policy functions. Then a Monte Carlo simulation is performed to compute some of the statistics. The algorithm to compute the policy functions can be summarized as follows,

1. Make an initial guess for the discount factor β and the level of bequests q_0. For β define $b = [b_1, b_2]$ (with $b1 < b2$), and let $\beta = (b_1 + b_2)/2$.

2. Starting from age J, and given prices consistent with the calibration, compute the value functions and associated policy functions that solve $(P1)$ by a single backward recursion.

3. Define the distribution of assets and shocks for the first cohort $j = 1$), and using the policy functions and the transition matrix for the shock, compute recursively the distribution of assets and shocks for ages $2, 3...J$.

4. Using the distribution of assets for all cohorts, calculate the implied levels of aggregate capital and bequests K_1, and q_1.

5. Compare K_1 with the level implied by the calibration $K*$. If convergence fails, adjust bequests with $q_0 = q_1$,and β by letting $b_1 = \beta$ if $K_1 < K*$, and $b_2 = \beta$ if $K_1 > K*$.

For both solution methods the grid size for assets is set to 5 -10% of average asset holdings, and the convergence criterion is set to .003. Convergence occurs generally in 7-9 iterations. Using the policy functions, we then simulate paths for 10.000 agents and compute the statistics. The original version of this algorithm is discussed in İmrohoroğlu et al. [1999].

A.2 Solution Method for the Deterministic OLG Model

We solve the deterministic version of the OLG model by a method presented in Rios-Rull [1999]. It is a 'shooting' method that uses the fact that the

Euler equation can be expressed as:

$$
\begin{aligned}
a_t = \quad & \{\frac{1+(1+r)(\beta(1+r))^{-1/\theta}}{1+r}\}a_{t+1} + \{-\frac{(\beta(1+r))^{-1/\theta}}{1+r}\}a_{t+2} \\
& +\{\frac{(w_{t+1}+q)(\beta(1+r))^{-1/\theta} - (w_t+q)}{1+r}\}
\end{aligned}
$$

1. Set technology and preference parameters, $\{a1 = a, \alpha, \beta, \theta, \delta\}$ and guess levels of aggregate capital K_0 and per capita bequests q_0.

2. Using K_0, and given the production function $F(K, L) = AK^{\alpha}L^{1-\alpha}$, find prices $\{r, w\}$.

3. Using the fact that $a_{t+1} = 0$, guess a value for a_T, and compute $\{a_t\}_{t=1}^{T}$ backwards using the Euler equation.

4. Check whether $a_1 = a$, otherwise go back to 3 and modify guess for a_T.

5. Calculate aggregate capital K_1 and per capita bequests q_1 using $\{a_t\}_{t=1}^{T}$. If convergence fails, set $q_0 = q$ and $K_0 = K_1$, and go back to 1.

Figure 1: Income Profile (Hubbard et al. [1994], 12-15 years of education)

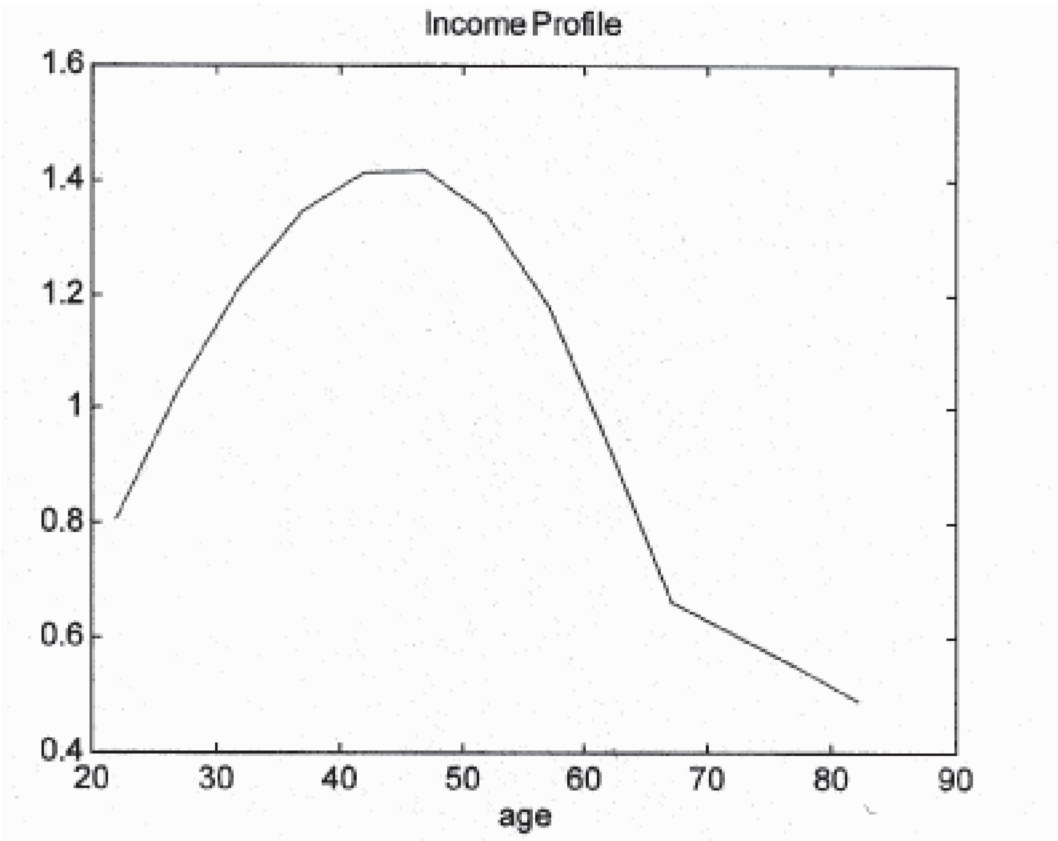

Figure 2: Age-Wealth Profile: Baseline Model

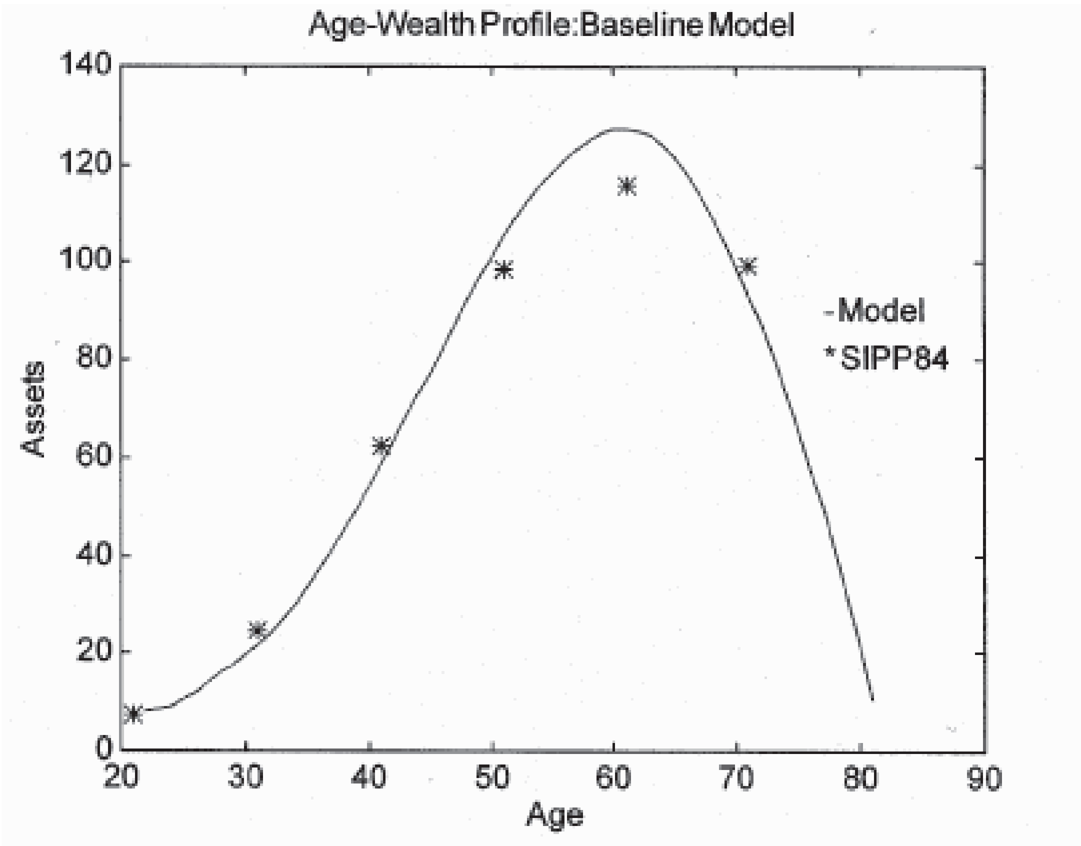

Figure 3: Age-Wealth Profile: Models 2 to 4

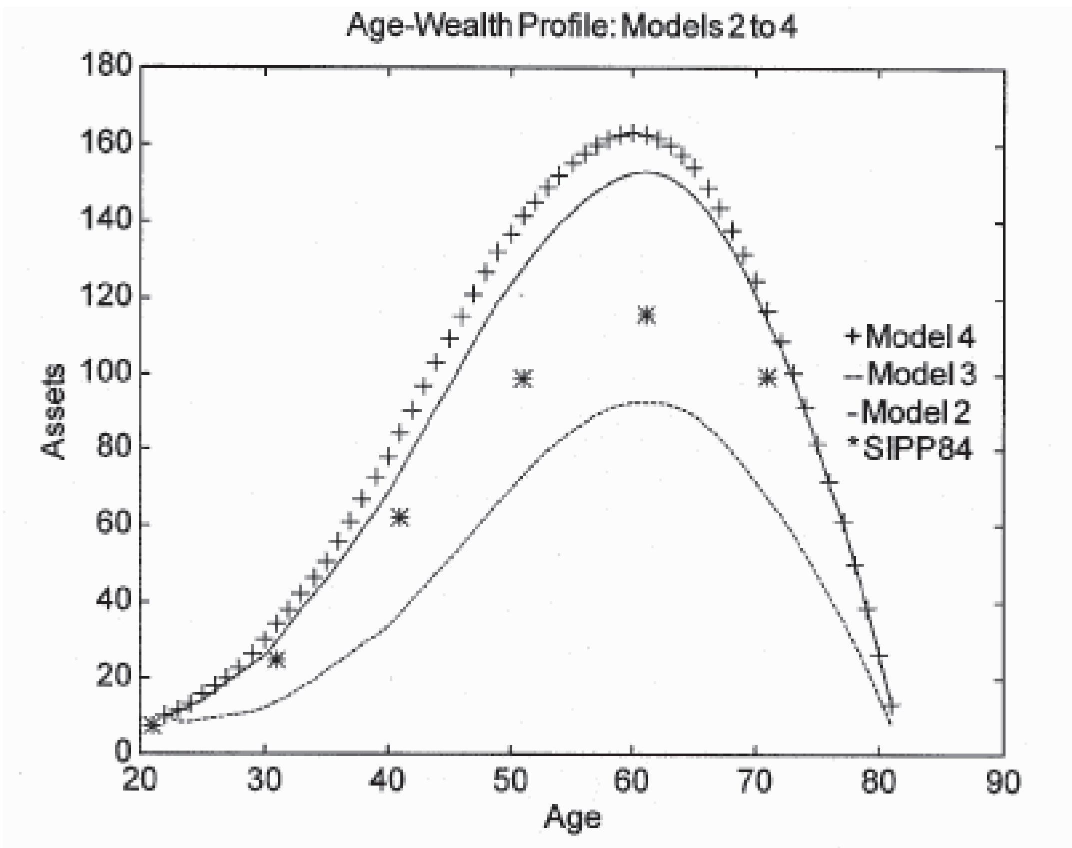

Figure 4: Age-Wealth Profile: Models 5 to 7

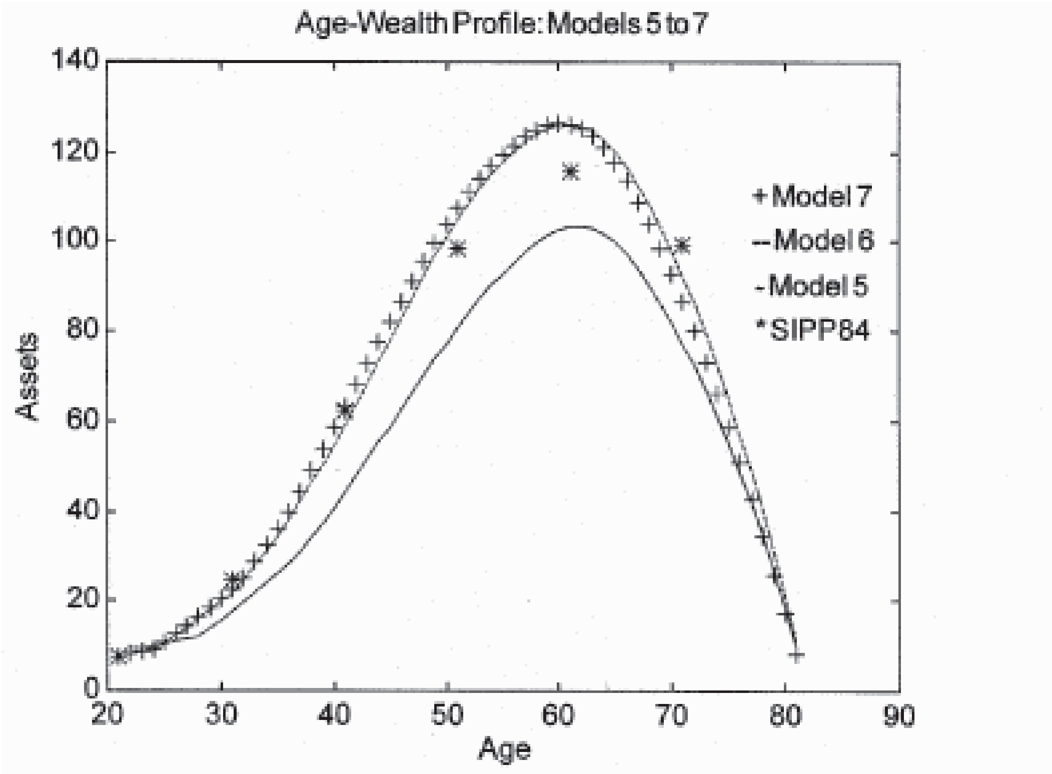

Table 1: Calibration

Model	Calibration Choices			Implied Parameters			Stochastic Process	
	S/Y	Interest R.	θ	β	δ	K/Y	ρ	σ_ϵ^2
Baseline	0.19	0.04	3	0.9815	0.0447	4.25	0.946	2.5%
Model 2	0.15	0.04	3	0.9983	0.0286	5.25	0.946	2.5%
Model 3	0.24	0.04	3	0.9556	0.08	3	0.946	2.5%
Model 4	0.19	0.03	3	1.011	0.0335	5.67	0.946	2.5%
Model 5	0.19	0.05	3	0.959	0.0559	3.4	0.946	2.5%
Model 6	0.19	0.04	5	0.9729	0.0447	4.25	0.946	2.5%
Model 7	0.19	0.04	1	0.9763	0.0447	4.25	0.946	2.5%

Figure 5: Average Propensities to Save

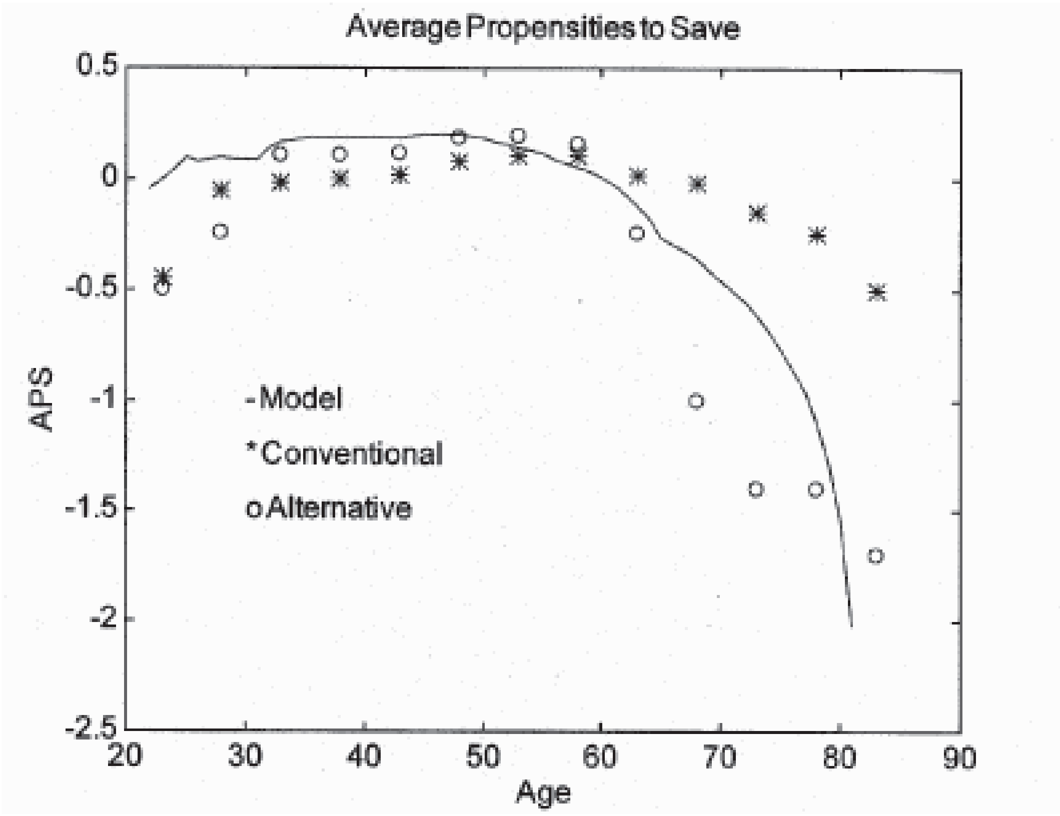

Table 2: Carroll-Samwick estimates for sensitivity of wealth holdings with respect to income risk

Model	Age<50	Age 1-82
C-S:Per. Var.	12.09	13.27
C-S:Tr. Var.	7.11	6.6
Baseline	23.76	18.69
Model 2	21.76	14.88
Model 6	8.61	3.99

Figure 6: Wealth/Income Profile: Baseline Model

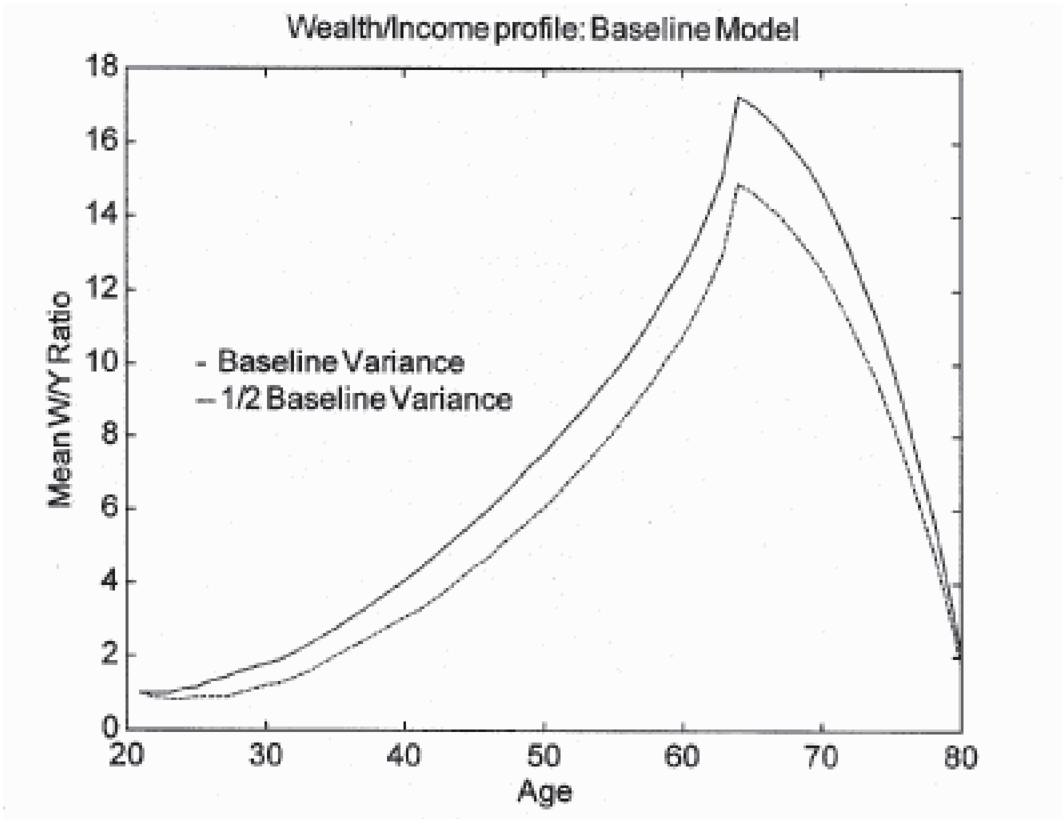

Figure 7: Wealth/Income Profile: Model 2

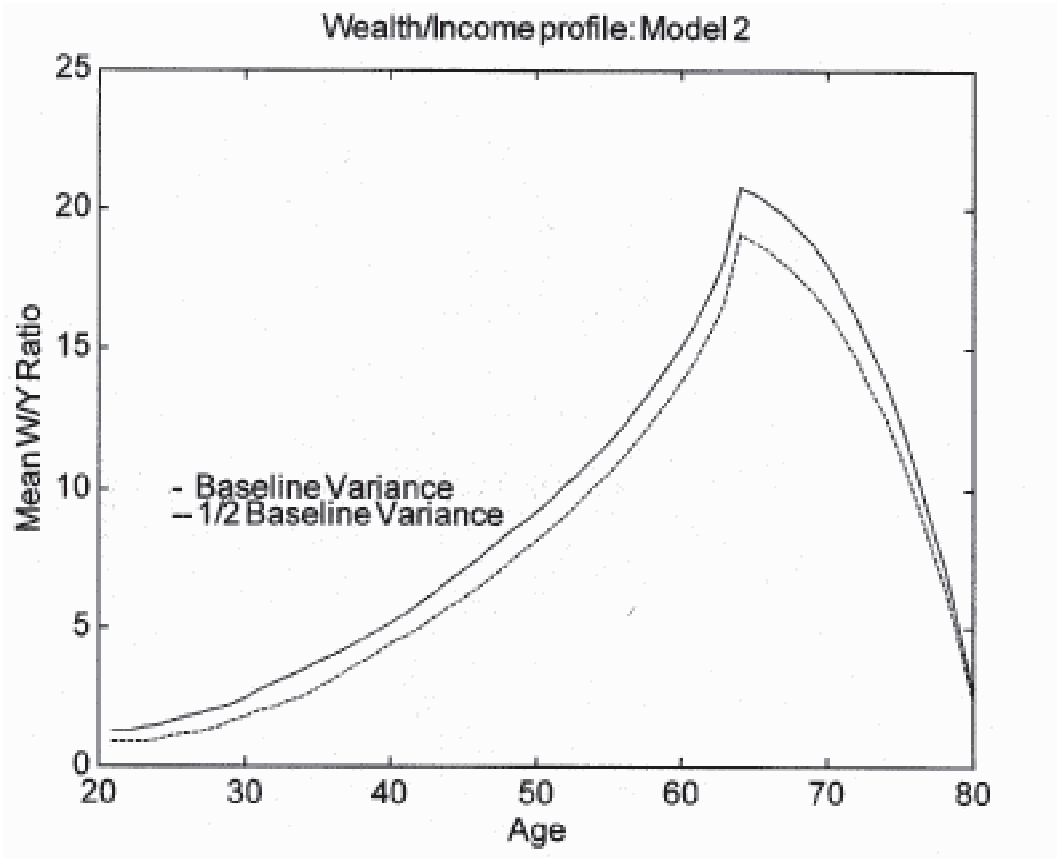

Figure 8: Wealth/Income Profile: Model 6

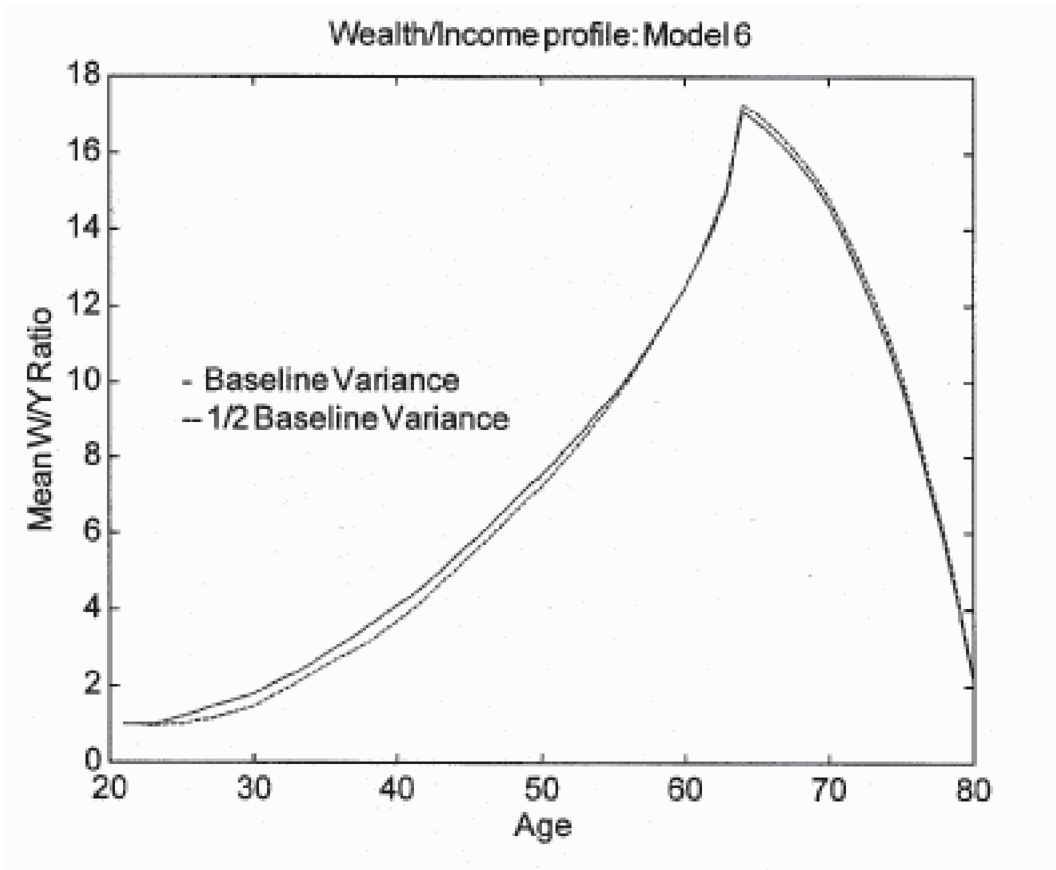

Figure 9: Age-Wealth Profile: Stochastic and Deterministic Economies

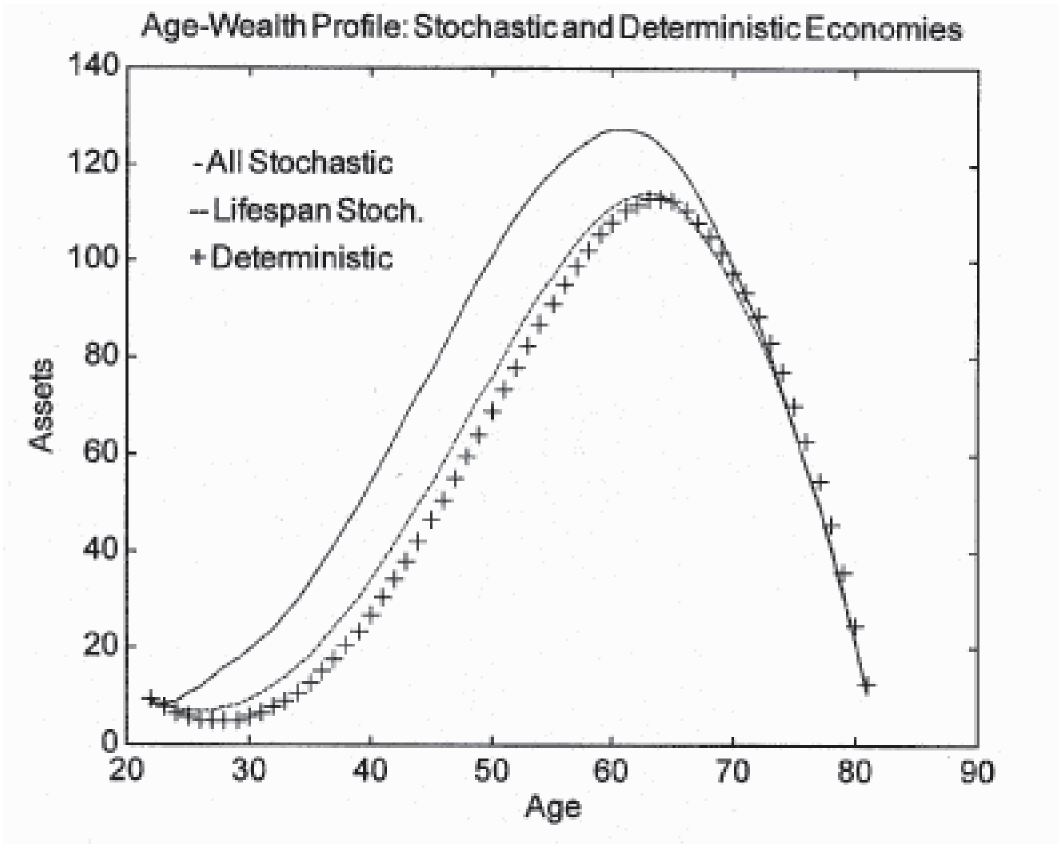

Table 3: General equilibrium estimates of precautionary wealth

Model	Calibration			Precautionary wealth (%)	
	S/Y	Interest R.	θ	Lifespan Uncertain	All Certain
Baseline	0.19	0.04	3	27.33	30.09
Model 2	0.15	0.04	3	23.42	26.75
Model 3	0.24	0.04	3	31.94	34.08
Model 4	0.19	0.03	3	22.38	25.63
Model 5	0.19	0.05	3	29.63	32.07
Model 6	0.19	0.04	5	49.63	50.71
Model 7	0.19	0.04	1	6.69	11.81

Table 4: Partial and general equilibrium estimates of precautionary wealth (%)

Model	Partial Eq. (1)	General Eq. (2)	$100 \times ((1) - (2))/(2)$
Baseline	44.92	30.09	49.29
Model 2	33.28	26.75	24.42
Model 3	76.03	34.08	123.11
Model 4	30.33	25.63	18.33
Model 5	60.36	32.07	88.25
Model 6	59.22	50.71	16.79
Model 7	39.34	11.81	233.19

Table 5: Reproducing results: % of wealth that is precautionary

Model	Lifespan Uncertain	All Certain
HSZ 1	68.29	71.93
HSZ 2	67.17	70.97
Skinner 1	47.86	50.87
Skinner 2	47.04	50.13

Table 6: Partial vs. general equilibrium effects

Model	% prec.	% Prec.	100 $\times$ ((1) $-$ (2))/(2)	% prec.	% Prec.	100 $\times$ ((3) $-$ (4))/(4)
	P.E. (1)	G.E. (2)		P.E. (3)	G.E. (4)	
HSZ	26.63	22.38	19	30.33	25.64	18.32
Skinner	41.62	27.63	50.67	44.6	31.06	43.59

Table 7: Selected Capital Output Ratios

	Interest Rate			
Saving Rate	.03	.04	.05	.06
.16	6.7	5	4	3.3
.18	6	4.5	3.6	3
.20	5.3	4	3.2	2.7
.22	4.7	3.5	2.8	2.3
.24	4	3	2.4	2